MATHEMATICAL MODEL OF THE INFLUENCE OF SOLID PARTICLE MASS CONCENTRATION IN MINE WATER ON THE WORKING ELEMENTS OF CENTRIFUGAL PUMPS

Khatamova Dilshoda Narmuratovna
Professor, Navoi State University of Mining
and Technologies, Navoi, Uzbekistan
E-mail: dilshodanarmuratovna@yandex.ru
Yuldasheva Mohinur Abduxakim kizi
PhD Student, Tashkent State Technical University named after Islam
Karimov, Tashkent, Uzbekistan
E-mail: yuldashevamohinur71@gmail.com

https://doi.org/10.5281/zenodo.20622102

Abstract. Mechanical impurities contained in mine water negatively affect the operating process of centrifugal pumps. Mine water contains solid particles of various sizes and concentrations, such as fragments of rock, quartz, and granite, which cause hydro-abrasive, corrosive, and cavitation wear of the pump working elements. As a result, the efficiency of the pump decreases, the pressure head drops, and energy consumption increases.

Keywords: Mine water, mechanical impurities, hydro-abrasive wear, pump impeller, solid particles, cavitation, concentration, energy consumption, pump head, efficiency coefficient.

Introduction.

In the mining industry, the process of mine water drainage is considered one of the most important and complex tasks of hydraulic systems. The composition of mine water is usually highly contaminated and contains a significant amount of mechanical particles of various sizes, including fragments of rocks, quartz, granite, coal, and ore. Their concentration varies within a wide range and, in some cases, may reach very high values. The presence of these solid particles significantly affects the operating performance of centrifugal pumps. In addition, cavitation processes develop in low-pressure zones, causing micro- and macro-scale damage on the surfaces of pump components. As a result, the hydraulic efficiency of the pump decreases, the pressure head drops, and energy consumption increases.

Literature review and methods.

The composition of mine water and its impact on pumping equipment have been studied by many local and foreign researchers. According to scientific sources, mechanical impurities present in mine water significantly affect the operational performance of pumps. Several studies have identified quartz and granite particles within the 0.1–0.3 mm size range as the most hazardous erosive factor. Such particles create micro-cuts on the surface of the impeller and accelerate the material wear process. Mathematical models developed in recent years have made it possible to describe wear intensity as a function of particle concentration, velocity, material hardness, and particle size. These models are of great importance in predicting the service life and energy efficiency of pumps.[1]

In this study, theoretical and practical methods were applied to investigate the wear process of centrifugal pumps operating with mine water. Initially, based on a literature review, the mechanisms of hydro-abrasive and corrosive wear were studied, and the main influencing factors were identified. Subsequently, a mathematical modeling approach was applied, in which the wear intensity was expressed as a function of particle density, diameter, velocity, concentration, and the properties of the impeller material. In addition, integration and statistical evaluation methods were used to determine the time-dependent variation of the wear process. Through this approach, the service life of the pump and its energy consumption values were determined.[2]

Results.

The processes of hydro-abrasive and cavitation wear occurring in pump impellers intensify most significantly at the inlet section of the blades and near the leading edge of the outlet side of the working surface (Figure 1). Cavitation develops in the low-pressure regions formed at the rear surface of the impeller in zone A, on the inner surface of the inlet edge in zone B, and in the clearance of the sealing edge in zone C. In addition, “crack-type” cavitation occurring in cavities formed as a result of flow contraction also develops. As a result, these processes mutually intensify each other, creating a complex hydrodynamic condition in the suction section of the pump and leading to an increase in the degree of additional rarefaction.[3]

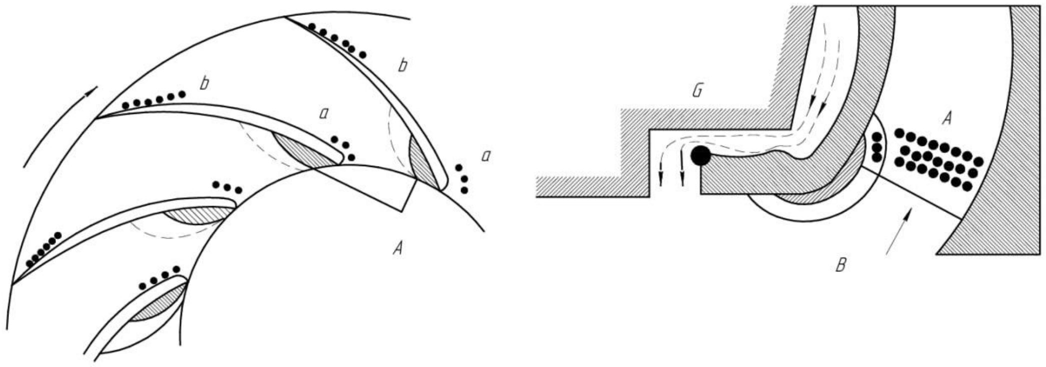

Figure 1. Intensive cavitation (A, B, C) and hydro-abrasive wear of mine pump impellers.

All major components and elements of centrifugal pumps along the flow path are most susceptible to wear processes, particularly the inlet blades of guide vanes, distance bushing connections, and especially the first-stage impeller.

According to practical data, the guaranteed service life of centrifugal pump components made of cast iron and steel is on average 5,000 hours. However, when the concentration of solid particles in mine water exceeds 0.5% and their size reaches 0.2 mm or larger, the wear of pump elements accelerates significantly, and the service life is reduced to approximately 2,500 hours. Under such wear conditions, the hydraulic performance of the pump also deteriorates; in particular, the nominal head may decrease by up to 8–10%.[4]

The wear intensity I_e generally depends on the following factors[5]:

$$I_e = f\ (\rho_z, d_z, C_z, v_z, \alpha, H_m, \rho_m),$$

In this expression, I_e is the wear intensity, expressed in kg/s or kg/(s·m^2); ρ_z is the particle density, kg/m^3; d_z is the particle diameter, m; C_z is the mass concentration of solid particles, mg/l; v_z is the particle impact velocity on the surface, m/s; α is the impact angle of the particle on the impeller surface, degrees; H_m is the hardness of the impeller material, N/m^2; and ρ_m is the density of the impeller material, kg/m^3.

The motion of a solid particle in a liquid medium, taking into account the interacting forces acting on it, is described by Newton’s Second Law:

$$m_p \frac{d\,\overrightarrow{v_z}}{dt} = \overrightarrow{F_d} + \overrightarrow{F_c} + \overrightarrow{F_g} + \overrightarrow{F_b} + \overrightarrow{F_{VM}} + \overrightarrow{F_{Saff}} \qquad (1)$$

In this expression, $m_z = \frac{\pi d_z^3}{6} \rho_z$is the particle mass, kg; F_d is the drag force, N; F_c is the centrifugal force, N; F_g is the gravitational force, N; F_b is the Archimedes force, N; F_{VM} is the added mass force, N; and F_{saff} is the Saffman lift force, N.

The wear intensity is expressed by the following formula:

$$I_e = K_W \frac{\rho_z}{H_m^m} \cdot C_{z0} \sum_{i=1}^{N} w_i v_{zi}^n f(\alpha_i) \left(1 + \frac{\lambda_{e,i}}{2}\right), \qquad (2)$$

In this expression, K_W is the generalized wear coefficient; n is the velocity exponent; $f(\alpha_i)$ is the function of the impact angle of the i-th particle fraction on the impeller surface; m is the hardness exponent; w_i is the weighting coefficient corresponding to the i-th particle; $\lambda_{e,i}$ is the empirical separation coefficient for the i-th particle (typically 0.5–2.0); C_{z0} is the average inlet concentration of particles; and N is the number of particle types (size fractions).

When the impeller thickness reaches the permissible limit value δ_{cheg}, the pump loses its operational capability. In the assessment of this process, the average wear intensity $(\overline{I_e})$is considered the main indicator. It becomes possible to determine the operating life of the pump and estimate the wear time:

$$T_{\text{res}} = \frac{\delta_{\text{cheg}} \rho_m H_m^m}{K_W \rho_z \cdot \frac{1}{A} \int_A C_{z0} \sum_{i=1}^{N} w_i v_{zi}^n f(\alpha_i) \left[1 + \lambda_{e,i} \left(\frac{r^2 - R_1^2}{R_2^2 - R_1^2}\right)\right] dA}, \qquad (3)$$

In this expression, T_{res} is the operating life of the pump, s; R_1, R_2 are the inner and outer radii of the impeller, m; r is the distance, m; A is the surface area (integration domain), m^2.

The total additional losses are expressed through hydraulic losses and losses due to clearance enlargement:

$$\Delta h_{\Sigma}(t) = \Delta h_f(t) + \Delta h_{cl}(t), \qquad (4)$$

$$\Delta h_{\Sigma}(t) = (k_1 v_s^2 + k_3) \cdot \frac{K_W \rho_z}{\rho_m H_m^m} \cdot C_{z0} \sum_{i=1}^{N} w_i v_{zi}^n f(\alpha_i) \left(1 + \frac{\lambda_{e,i}}{2}\right) \cdot t, \qquad (5)$$

In this expression, $\Delta h_f(t)$ is the additional hydraulic loss, m; $\Delta h_{\text{cl}}(t)$ is the loss due to clearance enlargement, m; $k_3 = k_{cl} k_e$ is the overall coefficient accounting for wear and flow effects, where k_{cl} is the flow loss coefficient through the clearance and k_e is the abrasive wear coefficient; $k_1 = \frac{k_\lambda k_\varepsilon L}{D^2 \cdot 2g}$ is a constant overall coefficient.

The useful head of the pump decreases as a result of wear:

$$H_t = H_0 - \Delta h_{\Sigma}(t), \qquad (6)$$

$$H_t = H_0 - (k_1 v_s^2 + k_3) \cdot \frac{K_W \rho_z}{\rho_m H_m^m} \cdot C_{z0} \sum_{i=1}^{N} w_i v_{zi}^n f(\alpha_i) \left(1 + \frac{\lambda_{e,i}}{2}\right) \cdot t, (7)$$

In this expression, H_t is the pump head after wear, m; H_0 is the pump head, m.

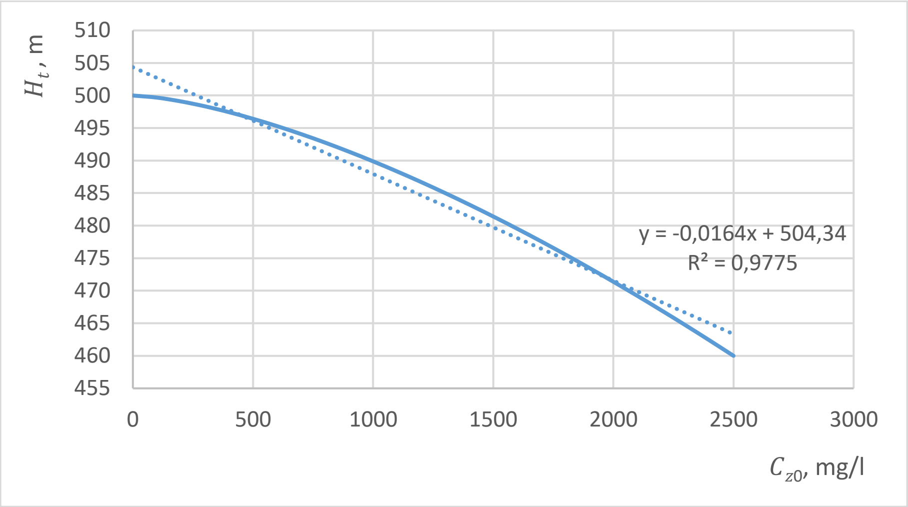

Figure 2. Dependence of pump head after wear (H_t)on the mass concentration of solid particles (C_{z0}).

The pump efficiency can be expressed in terms of head losses:

$$\eta_t = \eta_0 \left(1 - \frac{\Delta h_{\Sigma}(t)}{H_0}\right), \qquad (8)$$

$$\eta_t = \eta_0 \left[1 - \frac{1}{H_0}(k_1 v^2 + k_3) \cdot \frac{K_W \rho_z}{\rho_m H_m^m} \cdot C_{z0} \sum_{i=1}^{N} w_i v_{zi}^n f(\alpha_i) \left(1 + \frac{\lambda_{e,i}}{2}\right) \cdot t\right], (9)$$

In this expression, η_t is the pump efficiency after wear; η_0 is the pump efficiency.

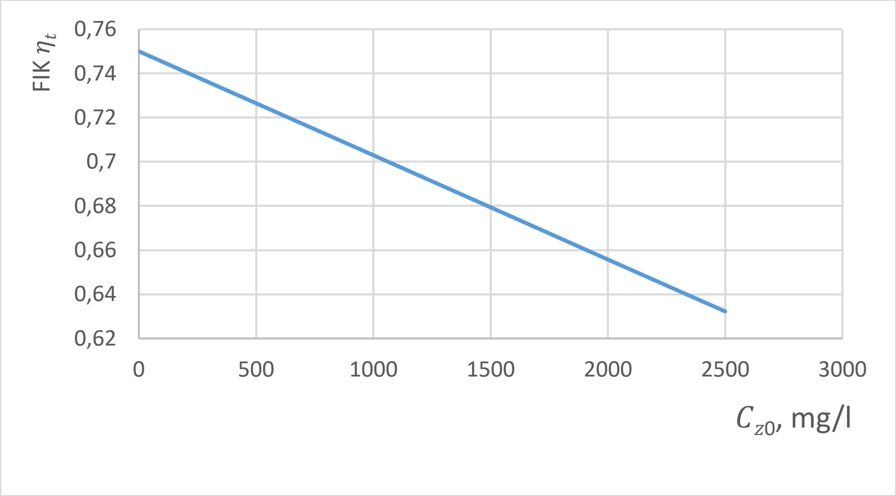

Figure 3. Dependence of pump efficiency after wear (η_t) on the mass concentration of solid particles (C_{z0}).

The initial power is $N_0 = \frac{\rho g Q H_0}{\eta_0}$. The power after wear (with respect to constant flow rate Q and head H_0) is expressed as:

$$N_t = \frac{\rho g Q H_0}{\eta_t}, \qquad (10)$$

In this expression, Q is the pump capacity, m^3/h; $g = 9.81$ is the gravitational acceleration, m/s^2.

Thus, the additional increase in power consumption is:

$$\Delta N(t) = N_t - N_0 = N_0 \frac{\Delta h_{\Sigma}(t)}{H_0}, \qquad (11)$$

$$\Delta N(t) = \frac{\rho g Q}{\eta_0} \cdot \frac{K_W \rho_z}{\rho_m H_m^m} \cdot (k_1 v^2 + k_3) \cdot C_{z0} \sum_{i=1}^{N} w_i v_{zi}^n f(\alpha_i) \left(1 + \frac{\lambda_{e,i}}{2}\right) \cdot t, \quad (12)$$

In this expression, $\Delta N(t)$ is the additional power consumption, kW; N_t is the power consumption after wear, kW; N_0 is the nominal pump power, kW.

Thus, the total power is:

$$N_t = N_0 + \frac{\rho g Q}{\eta_0}\cdot\frac{K_w\rho_z}{\rho_m H_m^m}\cdot(k_1v^2+k_3)\cdot C_{z0}\sum_{i=1}^{N} w_i v_{zi}^n f(\alpha_i)\left(1+\frac{\lambda_{e,i}}{2}\right)\cdot t,(13)$$

The total energy consumption required to pump 1 m^3 of liquid using the pump is determined as follows:

$$E_t = \frac{N_t}{Q}, \qquad (14)$$

$$E_t = \frac{1}{Q}\left[N_0 + \frac{\rho g Q}{\eta_0}\cdot\frac{K_w\rho_z}{\rho_m H_m^m}\cdot(k_1v^2+k_3)\cdot C_{z0}\sum_{i=1}^{N} w_i v_{zi}^n f(\alpha_i)\left(1+\frac{\lambda_{e,i}}{2}\right)\cdot t\right],(15)$$

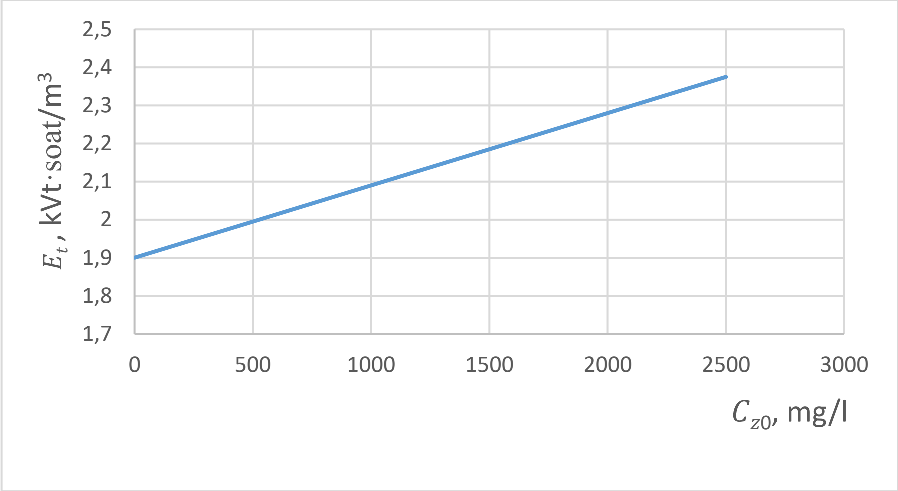

Figure 4. Dependence of the total energy consumption required to pump 1 m^3 of liquid (E_t) on the mass concentration of solid particles (C_{z0}).

As can be seen from the formulas, when the mass concentration of solid particles C_{z0}increases, the particle size d_z becomes larger, and the flow velocity term v^n increases, the pump service life, pump head, and pump efficiency decrease. Consequently, the total energy consumption required to pump 1 m^3 of liquid increases.

Conclusion.

When mine pumps operate under conditions of transporting turbid mine water, a reduction in the generated head is observed compared to operation with clean and clarified water. This decrease is associated with an increase in the relative density of mine water, which in turn is determined by the concentration of solid particles in the liquid. In addition, an increase in the amount of solid particles in mine water leads to higher power consumption of the pump drive, since the relative density of the fluid also increases. The presence of solid particles reduces the energy efficiency of the pump, and the degree of efficiency loss is directly dependent on the concentration of mechanical impurities in mine water.

References:

1.Ломакин В. О. Центробежные и осевые насосы. Москва: Машиностроение, 2010.

2.Xatamova D.N., Yuldasheva M.A. Nasos oqim yuzalarining shaxta suvi tarkibidagi qattiq zarrachalar sababli yemirilishini tadqiq qilish // Sanoatda raqamli texnologiyalar. Qarshi. 2026. 1-4 b. ISSN: 3030-3214.

3.Karassik I.J., Messina J.P., Cooper.P. Pump Handbook. USA. McGraw-Hill. 2008.-1789 p.

4.Stepanoff A.J. Centrifugal and Axial Flow Pump Theory, Design and Application. 1991. -462 p.

5.Gulich J. F. Centrifugal Pumps. Springer, 2014..