SOCIAL NETWORKS: EFFECTIVE METHODS OF DIVIDING INTO TWO AND THREE GROUPS

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Abstract. "Social networks: effective methods for dividing into two and three groups" is an objective look at current methods for analyzing social networks in order to identify subgroups or communities. The article examines in detail the use of various algorithms to determine the most clearly defined communities within social networks. Particular attention is paid to the importance of dividing into two or three groups to identify polarization of opinions or interest groups. The article also discusses the limitations of these methods related to the diversity of social network structures and the need for significant computing resources for large networks. In conclusion, the importance of updating and flexibility of approaches to analyzing social networks, taking into account the dynamic development of this area.

Keywords. Social networks. Social network analysis. Division into groups. Community detection methods. Algorithms. Polarization of opinions. Interest groups. Computing resources. Structure of social networks. The digital age. Maximum likelihood.

Introduction

The dynamics of social networks do not stand still; they are constantly changing and developing. As part of these changes, more and more specialized communities are emerging, bringing together people with common interests, goals or activities.

Communities on social networks are groups of users exchanging information on some common topic. This could be a community of consumers of certain products, fans of a popular group, professionals in a certain field, etc. The process of community formation usually occurs spontaneously. So-called "hubs" or "nucleus of the community" emerge, which over time attracts more and more participants. This process is ubiquitous and natural across all social media platforms, from Facebook and Instagram to professional platforms like LinkedIn. To identify and analyze communities, special algorithms are used that track user activity, their interaction with each other and common interests. All this allows you to notice clusters of users in certain "nodes" and highlight communities.

Communities play an important role in business and marketing - they allow brands to more effectively interact with their target audience, build trust with customers, receive feedback and even attract new consumers. With an understanding of how social media communities are formed and how they can be used commercially, brands have a powerful tool to grow their business.

The division of social networks into communities is a pressing problem in the fields of social sciences, computer science, and statistical physics. This is highlighted in many studies, including the following:

The author's work [1], presented in this review, explains various methods for detecting social and social networks, which are based on the network structure that dynamically interacts and recognizes participants. Particular attention is paid to methods that cope with the detection of overlapping and hierarchical communities. In the author's work [2], presented in this review, the study examines the structure of the Internet community. Newman shows that the Internet is a tremendous example of a network in which complex community structures can be identified, analyzed, and used to improve services and offerings to users.

In the author's work [3], presented in this review, the Authors emphasize that similarity, or the property that similar people are related to each other, creates strong communities in social networks. They discuss how this trend may influence the dynamic evolution of social networks and their overall structure.

[4] The book discusses in detail various metrics and methods for analyzing social networks, including community detection. He emphasizes the relevance of this problem for understanding the principles of organization and functioning of social networks. [5] highlights the versatility of community discovery methods that cross boundaries between social and biological networks. The authors present a method that analyzes network structure and identifies barriers between communities. [6] in How Digital Customer Communities Build Your Business by Larry Weber Weber argues that communities and social media can be used to strengthen a brand, delight customers, and expand business opportunities.

Thus, these studies highlight the relevance and importance of discovering and analyzing communities on social networks in various fields, including computer science, sociology, and marketing.

Main part. Determine the types of social networks when dividing them into three communities

Given a social network with n vertices, consider the problem of dividing this social network into 3 communities.

For a social network to be divided into 3 communities, the number of vertices must be at least 6 ($n \geq 6$). When n=6, the number of divisions of the social network into 3 groups is 1, i.e. (2;2;2). Below we write a certain number of divisions depending on the number of vertices:

TABLE 1. Certain divisions in the number of vertices

Number of vertices	group division	Number of divisions
$n = 6$	(2;2;2)	1
$n = 7$	(2;2;3)	1
$n = 8$	(2;2;4), (2;3;3)	2
$n = 9$	(2;2;5), (2;3;4), (3;3;3)	3
$n = 10$	(2;2;6), (2;3;5), (2;4;4), (3;3;4)	4
$n = 11$	(2;2;7), (2;3;6), (2;4;5), (3;3;5), (3;4;4)	5
$n = 12$	(2;2;8), (2;3;7), (2;4;6), (2;5;5), (3;3;6), (3;4;5), (4;4;4)	7

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Let us write the division of commands by the number of vertices in the first command as follows:

TABLE 2. Dividing teams by the number of vertices in the first team

Number of vertices	2	3	4	5	Amount
$n = 6$	1	0	0	0	1
$n = 7$	1	0	0	0	1
$n = 8$	2	0	0	0	2
$n = 9$	2	1	0	0	3
$n = 10$	3	1	0	0	4
$n = 11$	3	2	0	0	5
$n = 12$	4	2	1	0	7
$n = 13$	4	3	1	0	8
$n = 14$	5	3	2	0	10
$n = 15$	5	4	2	1	12
$n = 16$	6	4	3	1	14
$n = 17$	6	5	3	2	16

When splitting a social network with n nodes into 3 communities, we can create a model to calculate the total number of splits.

When dividing a social network into 2 communities, we use the $c(n) = \frac{n}{2} - \left(\frac{3}{2}\right)^{\frac{1-(-1)^n}{2}}$ formula to determine their number.

Number of splits with two vertices in the first command:

$$c_1(n) = \frac{n-2}{2} - \left(\frac{3}{2}\right)^{\frac{1-(-1)^n}{2}}$$

Number of partitions with 3 vertices in the first command:

$$c_2(n) = \frac{n-5}{2} - \left(\frac{3}{2}\right)^{\frac{1-(-1)^{n-5}}{2}} = \frac{n-5}{2} - \left(\frac{3}{2}\right)^{\frac{1+(-1)^n}{2}}$$

where $n \geq 9$.

Number of splits from 4 vertices in the first command:

$$c_3(n) = \frac{n-8}{2} - \left(\frac{3}{2}\right)^{\frac{1-(-1)^{n-8}}{2}} = \frac{n-8}{2} - \left(\frac{3}{2}\right)^{\frac{1-(-1)^n}{2}}$$

where $n \geq 12$.

Number of partitions with k vertices in the first command:

$$c_k(n) = \frac{n-(3k-1)}{2} - \left(\frac{3}{2}\right)^{\frac{1-(-1)^{n-(3k-1)}}{2}}$$

where $k = \left[\frac{n}{3}\right] - 1$ (The number $[n]$ is the integer part of n).

We calculate the number of all divisions:

$$c(n) = c_1(n) + c_2(n) + \ldots + c_k(n)$$

We introduce the permutations $\tau = \left(\frac{3}{2}\right)^{\frac{1-(-1)^n}{2}}$ and $\tau' = \left(\frac{3}{2}\right)^{\frac{1+(-1)^n}{2}}$. Then the formulas will look like this:

$$c_1(n) = \frac{n-2}{2} - \tau$$

$$c_2(n) = \frac{n-5}{2} - \tau'$$

…

$$c_k(n) = \begin{cases} \frac{n-(3k-1)}{2} - \tau, & If\ k\ is\ an\ even\ number \\ \frac{n-(3k-1)}{2} - \tau', & If\ k\ is\ an\ odd\ number \end{cases}$$

$$c(n) = \sum_{i=1}^{k} c_i(n) = \frac{kn}{2} - \frac{k(3k+1)}{4} - \frac{2k-1+(-1)^k}{4}\tau - \frac{2k+1-(-1)^k}{4}\tau' =$$

$$= \frac{kn}{2} - \frac{k(3k+1)}{4} - \frac{k}{2}\tau - \frac{k}{2}\tau' - \frac{-1+(-1)^k}{4}\tau - \frac{1-(-1)^k}{4}\tau' =$$

$$= \frac{k}{2}\left(n - \frac{3k}{2} - \frac{1}{2} - (\tau+\tau')\right) + \frac{1-(-1)^k}{4}(\tau - \tau')$$

where $\tau + \tau' = \frac{5}{2}$ и $\tau - \tau' = \begin{cases} \frac{1}{2}, & If\ n\ is\ an\ even\ number \\ -\frac{1}{2}, & If\ n\ is\ an\ odd\ number \end{cases}$ using the equations we get

$$c(n) = \frac{k}{2}\left(n - \frac{3k}{2} - 3\right) + \frac{1-(-1)^k}{8}(-1)^n \quad (1)$$

Division Of Social Networks Into Two And Three Groups

Dividing a social network into communities is a key point in social network analysis. From the point of view of a systematic approach, the number of communities on the network is not particularly important - it can be two, three or even more. It is important to understand that regardless of the number of communities, the main purpose of partitioning is to identify groups of nodes (or users) that have greater connectivity within the group and less connectivity between groups.

Dividing a social network into two communities is usually used when the goal of the analysis is to divide users into two separate groups according to a certain attribute or criterion. Such a division can be useful, for example, when studying political views (liberals versus conservatives), consumption preferences (coffee drinkers versus tea drinkers), etc.

Dividing a social network into three communities is usually done when there is a need to distinguish three different groups of users. A larger number of groups may allow us to characterize the structure of a social network in more detail and identify more complex patterns of interaction between users. For example, in a corporate network, dividing into three communities can help identify three different departments or teams within the organization.

However, it should be noted that the decision on the number of communities should be based on the specific objectives of the study and the nature of the data being collected. A larger number of communities can provide a more detailed understanding of a social network, but it can also increase the complexity of analyzing and interpreting the results.

Dividing a social network into two and three communities introduces properties that differ from each other in several related aspects:

1. **Complexity of analysis:** The more communities you want to identify on the network, the more complex the analysis becomes. When dividing into three communities, there is greater complexity compared to dividing into two, since it is necessary to take into account more interactions and relationships between network participants.

2. **Information granularity:** Dividing into three communities provides a finer granularity of information about the interactions and connections between network participants, while dividing into two communities can provide a more generalized picture.

3. **Data interpretation:** When a social network is divided into two communities, data interpretation is usually easier since participants either belong to one group or the other. But when divided into three communities, there is an additional group to count, which makes interpreting the data more difficult.

4. **Granularity:** Three communities provide a more accurate view of the nodes and their relationships in the network because they can be divided into smaller, specific groups. Isolating just two communities results in a coarser division that may not reveal all the nuances of node interactions.

5. **Computational Requirements:** Dividing into three communities typically requires more computational resources and processing time because more data needs to be analyzed and clustered.

Depending on the final goal and specifics of the study, different approaches may be more or less preferable.

Numerical experiments

Given a social network with 9 vertices and 16 edges.

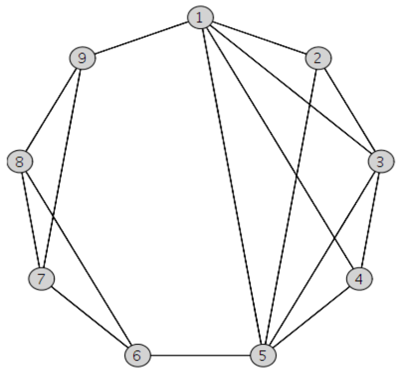

Figure 1. Social network with 9 vertices and 16 edges.

Let's divide the social network into 2 groups. The type of divisions can be determined using the formula from [7].

$$c(9) = \frac{9}{2} - \left(\frac{3}{2}\right)^{\frac{1-(-1)^9}{2}} = 4{,}5 - 1{,}5 = 3$$

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There are 3 types of splitting: (4,5), (3,6) and (2,7). We calculate the maximum likelihood for each partition.

1) a partition of type (4,5) is the largest maximum likelihood in the following cases:

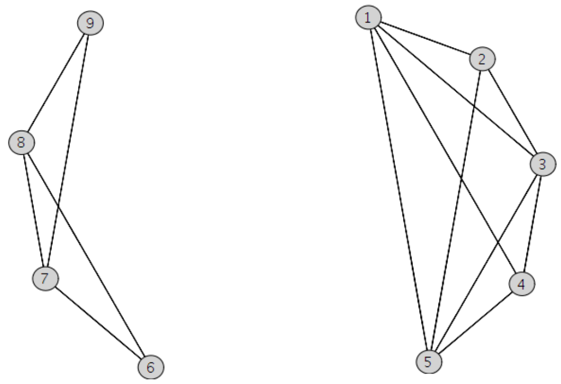


Figure 2. partition type (4,5)

$$l_{\Pi} = 14 ln p_{in} + 2 ln(1 - p_{in}) + 2 ln p_{out} + 18 ln(1 - p_{out})$$

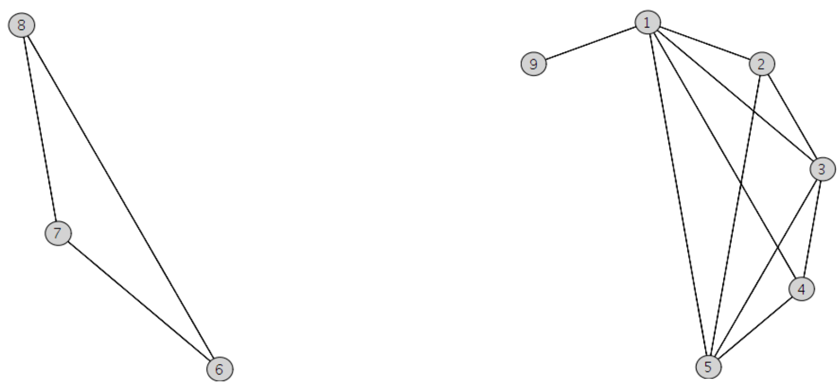
We differentiate it by p_{in} and p_{out} and equate it to zero. Having resolved the resulting system of equations, we find probability estimates and the value of the likelihood function, which gives

$$p_{in} = \frac{7}{8}, p_{out} = \frac{1}{10}$$

then

$$l_{\Pi}(p_{in}; p_{out}) = -12.52998205$$

2) a partition of type (3.6) is the largest maximum likelihood in the following



cases:

Figure 2. partition type (3,6)

$$l_{\Pi} = 13 ln p_{in} + 5 ln(1 - p_{in}) + 3 ln p_{out} + 15 ln(1 - p_{out})$$

We differentiate it by p_{in} and p_{out} and equate it to zero. Having resolved the resulting system of equations, we find probability estimates and the value of the likelihood function, which gives

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$$p_{in} = \frac{13}{18}, p_{out} = \frac{1}{6}$$

then

$$l_{\Pi}(p_{in}; p_{out}) = -18.74526219$$

3) a partition of type (2,7) is the largest maximum likelihood in the following cases

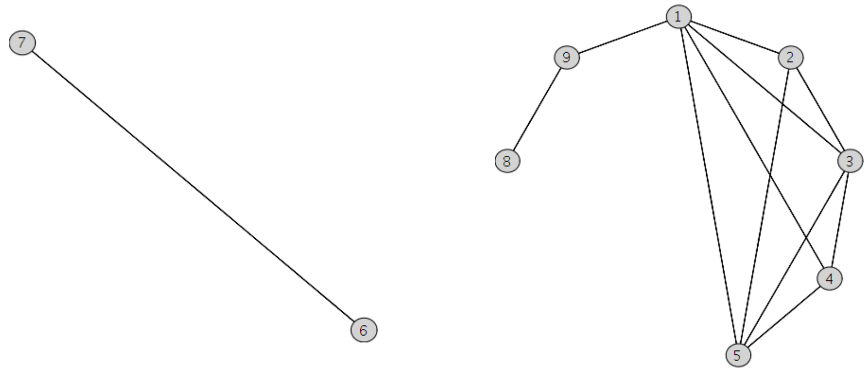

Figure 3. partition type (2,7)

$$l_{\Pi} = 12lnp_{in} + 10ln(1 - p_{in}) + 4lnp_{out} + 10ln(1 - p_{out})$$

We differentiate it by p_{in} and p_{out} and equate it to zero. Having resolved the resulting system of equations, we find probability estimates and the value of the likelihood function, which gives

$$p_{in} = \frac{6}{11}, p_{out} = \frac{2}{7}$$

then

$$l_{\Pi}(p_{in}; p_{out}) = -23.53397750$$

It can be seen that when dividing the above social network into 2 communities, the maximum likelihood probability between them reaches its greatest value when dividing like (4.5) $l_{\Pi}(p_{in}; p_{out}) = -12.52998205$.

Now let's divide the social network into 3 groups.

According to the above formula (1), $c(9) = 3$, these are (2;2;5), (2;3;4) and (3;3;3)

4) a partition of type (2,2,5) is the largest maximum likelihood in the following cases:

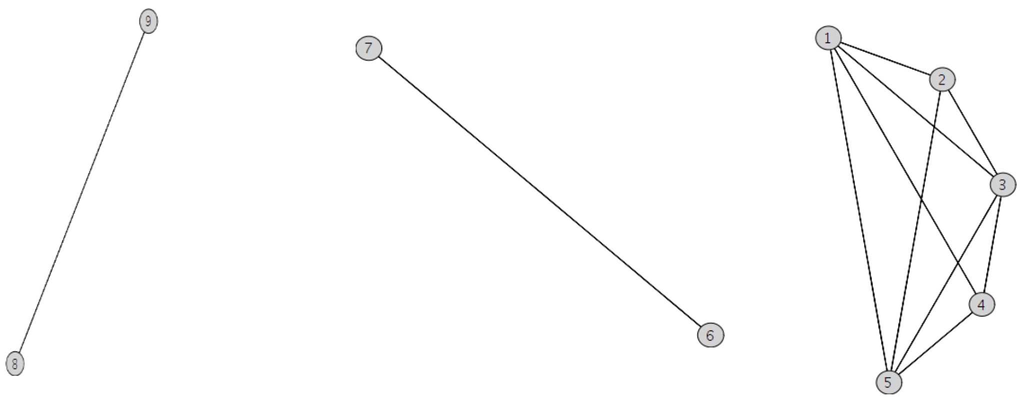

Figure 4. partition type (2,2,5)

$$l_{\Pi} = 11lnp_{in} + ln(1 - p_{in}) + 5lnp_{out} + 19ln(1 - p_{out})$$

We differentiate it by p_{in} and p_{out} and equate it to zero. Having resolved the resulting system of equations, we find probability estimates and the value of the likelihood function, which gives

$$p_{in} = \frac{11}{12}, p_{out} = \frac{5}{24}$$

then

$$l_{\Pi}(p_{in}; p_{out}) = -15.72379356$$

5) a partition of type (2,3,4) is the largest maximum likelihood in the following cases:

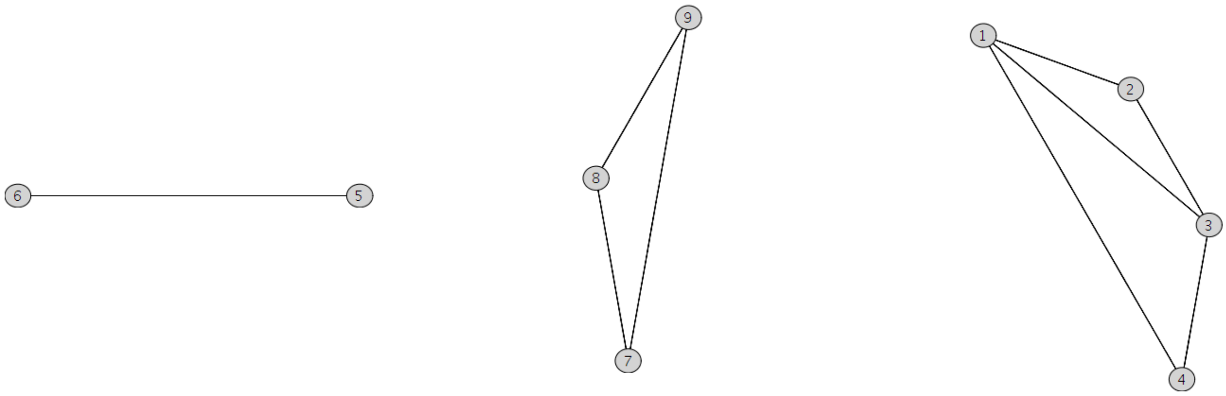

Figure 5. partition type (2,3,4)

$$l_{\Pi} = 9lnp_{in} + ln(1 - p_{in}) + 7lnp_{out} + 19ln(1 - p_{out})$$

We differentiate it by p_{in} and p_{out} and equate it to zero. Having resolved the resulting system of equations, we find probability estimates and the value of the likelihood function, which gives

$$p_{in} = \frac{9}{10}, p_{out} = \frac{7}{26}$$

then

$$l_{\Pi}(p_{in}; p_{out}) = -18.39562808$$

6) a partition of type (3,3,3) is the largest maximum likelihood in the following cases:

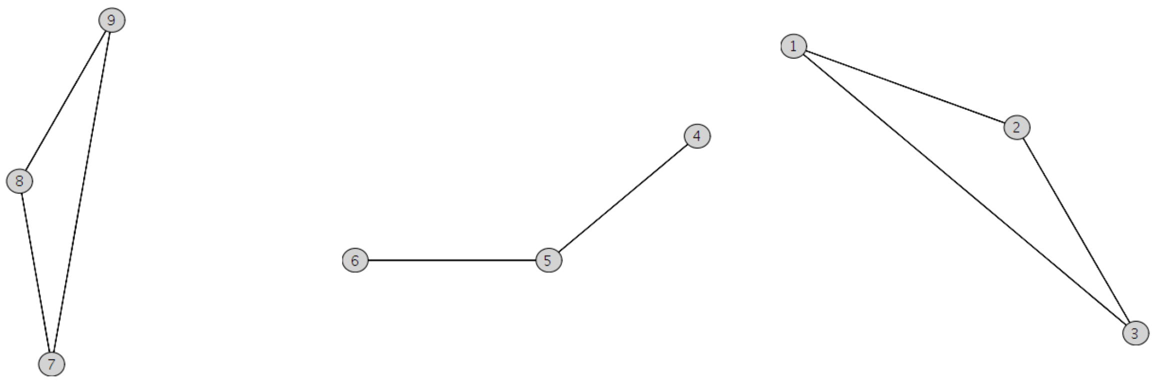

Figure 6. partition type (3,3,3)

$$l_{\Pi} = 8lnp_{in} + ln(1 - p_{in}) + 8lnp_{out} + 19ln(1 - p_{out})$$

We differentiate it by p_{in} and p_{out} and equate it to zero. Having resolved the resulting system of equations, we find probability estimates and the value of the likelihood function, which gives

$$p_{in} = \frac{8}{9}, p_{out} = \frac{8}{27}$$

then

$$l_{\Pi}(p_{in}; p_{out}) = -19.54721131$$

it can be seen that when dividing the above social network into 3 communities, the maximum likelihood probability between them reaches its greatest value when dividing like (2,2,5) $l_{\Pi}(p_{in}; p_{out}) = -15.72379356$.

when dividing the social network into 2 and 3, the maximum likelihood probability among them arises when dividing the social network into 2 communities. Since the maximum likelihood probability in division of type (4,5) reaches the highest value $l_{\Pi}(p_{in}; p_{out}) = -12.52998205$.

Conclusion

Community recognition and detection play an important role in understanding the structure and dynamics of social networks. Methods that classify social networks into two or three groups offer a robust approach to the study of social interaction and information exchange.

However, it is important to remember that there is no universal method that is ideal for every social network or scenario. When dividing networks into groups, researchers must take into account the specific features and structure of each individual network. For example, some networks have complex or hierarchical structures in which division into two or three groups would not adequately or faithfully reflect the relationships within those communities.

In addition, the application of community analysis and distribution methods can be computationally intensive, especially in the case of large networks. Therefore, strategies for optimization and efficient use of resources are also important aspects in this field.

Finally, it is worth noting that given the rapid growth and development of digital technologies and social networks, community detection methods will continue to evolve in response to new challenges and needs. Therefore, a critical and updated understanding of these methods is essential for effective and relevant social media research. The field of social networks is a dynamic and promising field, transforming and providing new approaches to understanding the study of social interactions and processes.

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