THE IMPORTANCE OF STUDYING WORD PROBLEMS IN THE SCHOOL MATHEMATICS COURSE

Abdullayev A. G.
"University of Economics and Pedagogy", in Andijan.
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Abstract

This article discusses the importance of mastering word problems in the school mathematics course. It presents methods for analyzing certain complex texts and constructing corresponding equations and inequalities. This work is intended for school teachers and students.

Keywords: school, root, equation, inequality, ratio, mixture.

Introduction

A country always needs specialists with deep and comprehensive knowledge of mathematics. Therefore, one of the urgent tasks of school mathematics teachers is to identify students with an aptitude for mathematical knowledge and provide them with deeper and broader knowledge through a specialized program.

A future engineer or economist must be able to express various processes in mathematical language. To achieve this, they must first be able to correctly and comprehensively formulate equations and inequalities (i.e., models) that correspond to complex word problems studied within the school curriculum.

Observations show that many students face significant difficulties when analyzing complex word problems in elementary mathematics and formulating the necessary equations and inequalities to solve them. Existing school textbooks do not sufficiently cover the methods and essence of solving such problems. This methodological guide aims to provide recommendations on how to correctly and fully utilize problem statements by examining some of these more complex problems. It is primarily intended for school teachers, students, and independent learners of mathematics.

Problems on Mixtures

Problem 1

There are three different mixtures composed of elements A, B, and C.

- The first mixture consists only of elements A and B, combined in a 3:5 ratio.
- The second mixture consists only of elements B and C, combined in a 1:2 ratio.
- The third mixture consists only of elements A and C, combined in a 2:3 ratio.

If these three mixtures are combined in a certain proportion, what ratio must they be mixed in to ensure that the resulting new mixture contains elements A, B, and C in a 3:5:2 ratio?

Solution

Let's assume that the amounts of elements A, B, and C in the new mixture are represented by x, y, and z, respectively. Our goal is to determine the ratio x:y:z. To set up the correct equations, we need to consider the following:

1. In the first mixture, each quantity consists of 3/8 A and 5/8 B.

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2. In the second mixture, each quantity consists of 1/3 B and 2/3 C.
3. In the third mixture, each quantity consists of 2/5 A and 3/5 C.
4. In the final new mixture, each quantity consists of 3/10 A, 5/10 B, and 2/10 C.

Thus, for the amount of element A in the new mixture, we can write the following equation:

$$\frac{3}{8}\mathbf{x} + \frac{2}{5}\mathbf{z} = \frac{3}{10}(\mathbf{x}+\mathbf{y}+\mathbf{z})$$

Similarly, for the amount of element B in the new mixture, we can write the following equation:

$$\frac{5}{8}\mathbf{x} + \frac{1}{3}\mathbf{y} = \frac{5}{10}(\mathbf{x}+\mathbf{y}+\mathbf{z})$$

By adding the corresponding equation for the amount of element C in the new mixture, we obtain the following system of equations:

$$\begin{cases}\frac{3}{8}\mathbf{x} + \frac{2}{5}\mathbf{z} = \frac{3}{10}(\mathbf{x}+\mathbf{y}+\mathbf{z})\\ \frac{5}{8}\mathbf{x} + \frac{1}{3}\mathbf{y} = \frac{5}{10}(\mathbf{x}+\mathbf{y}+\mathbf{z})\\ \frac{2}{3}\mathbf{y} + \frac{3}{5}\mathbf{z} = \frac{2}{10}(\mathbf{x}+\mathbf{y}+\mathbf{z})\end{cases}$$

Here, we obtain a system of three equations with three unknowns. However, in reality, only two of these equations are independent. This is because if we subtract the sum of the first two equations from the identity **x + y + z = x + y + z** the third equation is derived.

Thus, from this system, we can only determine the ratio **x:y:z.** Eliminating **x** from the first and second equations, we find that **y=2z** (i.e., **y:z = 2:1**). Substituting this relation into one of the equations, we obtain **x:z = 20:3**, leading to:

x:y:z = 20:6:3.

Problem 2

There are three different alcohol mixtures, where the alcohol content by weight follows a geometric progression.

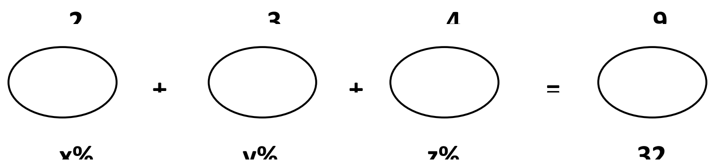
- If these mixtures are combined in a **2:3:4** ratio, the resulting mixture has an alcohol concentration of **32%.**
- If these mixtures are combined in a **3:2:1** ratio, the resulting mixture has an alcohol concentration of **22%.**

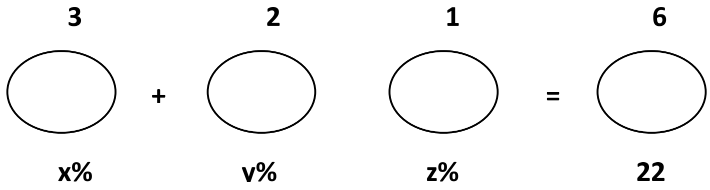
Find the concentration of the first mixture.

Solution

Let the alcohol concentration of the first, second, and third mixtures be **x%**, **y%**, and **z%**, respectively.

Based on these given ratios, we construct a system of equations as follows:





$$\begin{cases} 2x + 3y + 4z = 9 \cdot 32 \\ 3x + 2y + z = 6 \cdot 22 \end{cases}$$

Since **x** , **y** and **z** are consecutive terms of a geometric progression, we assume:

$y^2 = xz$

Thus, we obtain the following system of equations:

$$\begin{cases} 2x + 3y + 4z = 288 \\ 3x + 2y + z = 132 \\ y^2 = xz \end{cases}$$

From the first two equations, we find:

$y = 48 - 2x$, $z = 36 + x$

Substituting these into the third equation, we obtain:

$x^2 - 76x + 768 = 0$

Solving this quadratic equation, we get the roots:

$x_1 = 12$, $x_2 = 64$

Since the second root does not satisfy the problem conditions (it results in a negative value for **y**), the solution to the problem is:

$x = 12\%$

Problem 3.

A container with a capacity of 729 liters was initially filled with pure acid. After removing **x** liters of acid, the same amount of pure water was added. Then, after thoroughly mixing the liquid in the container, another **x** liters was removed, and the same amount of pure water was added again. This process was repeated six times. If, after the sixth iteration, the amount of pure acid remaining in the container was 64 liters, find the value of **x**.

Solution: After the first removal of **x** liters of acid and replacing it with water, the amount of pure acid left in the container is: **(729 – x)**

In other words, each liter of the mixture in the container now contains: $\frac{729-x}{729}$

Therefore, when another **x** liters is removed in the second step, the amount of acid removed is:

$x \cdot \frac{729-x}{729}$

Thus, the remaining amount of pure acid in the container is:

$$729 - x - x \cdot \frac{729-x}{729} = \frac{(729-x)^2}{729}$$

After the container is refilled with water, each liter of the mixture contains:

$\frac{(729-x)^2}{729^2}$ parts of pure acid. Therefore, after removing **x** liters of liquid for the third time, the remaining amount of pure acid in the container is:

$\frac{(729-x)^2}{729} - x \cdot \frac{(729-x)^2}{729^2} = \frac{(729-x)^3}{729^2}$

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Following this pattern, after repeating the process six times, the amount of pure acid left in the container can be expressed as: $\frac{(729-x)^6}{729^5}$

Thus, to determine **x**, we set up the equation: $\frac{(729-x)^6}{729^5} = 64$

Solving this equation, we find that, $x = 243$.

Recommended Problems for Independent Study

Motion Problems

Problem 4: Two friends set off at the same time from point A to point B, one on a bicycle and the other on foot. At some point, the first friend leaves the bicycle and continues on foot to point B. The second friend, upon reaching the bicycle, rides it for the remaining distance and arrives at point B at the same time as the first friend.

On their way back from point B to point A, they follow the same pattern. However, this time, the first friend rides the bicycle **one kilometer farther** than before. As a result, the second friend arrives **21 minutes later** than the first.

It is known that both friends ride the bicycle at **20 km/h**, and when walking, the first friend takes **3 minutes less per kilometer** than the second friend. Determine their walking speeds.

Solution

Let the walking speeds of the first and second friend be **x** km/h and **y** km/h, respectively, and let the total distance from A to B be **S** km. Assume the first friend leaves the bicycle at **b** km from A.

To form the equations, we express the time taken to travel from A to B:

- The first friend rides **b** km by bicycle and walks the remaining **(S – b)** km and for this he spends **b/20 + (S-b)/x hours.**
- Similarly, the second friend when travelling from A to B spends **b/y + (S-b)/20** hours.

Since they arrive at B at the same time, we equate these expressions:

$$\frac{b}{20} + \frac{S-b}{x} = \frac{b}{y} + \frac{S-b}{20}$$

On the return journey (from B to A):

- The first friend rides (**b + 1) km** and walks the remaining (**S−b−1**) km, and for this spends (**b+1)/20 + (S-b-1)/x** hours.
- The second friend spends (**b+1)/y + (S-b-1)/20** hours.

The second friend arrives **21 minutes later**, so:

$$\frac{b+1}{20} + \frac{S-b-1}{x} = \frac{b+1}{y} + \frac{S-b-1}{20} - \frac{7}{20}$$

One more equation can be written considering their time spent on 1 km:

$$\frac{1}{y} - \frac{1}{x} = \frac{1}{20}$$

Thus, we have fully utilized the given conditions of the problem and obtained the following system of equations:

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$$\begin{cases} \frac{b}{20} + \frac{S-b}{x} = \frac{b}{y} + \frac{S-b}{20} \\ \frac{b+1}{20} + \frac{S-b-1}{x} = \frac{b+1}{y} + \frac{S-b-1}{20} - \frac{7}{20} \\ \frac{1}{y} - \frac{1}{x} = \frac{1}{20} \end{cases}$$

This is a system of three equations with four unknowns, and at first glance, it may seem impossible to determine the values of **x** and **y.** However, upon closer examination, by subtracting the first equation from the second, we eliminate the variables **b** and **S**, resulting in the following equation:

$$\frac{1}{y} + \frac{1}{x} = \frac{9}{20}$$

Solving this equation together with the third equation, we will find that:

y = 4 km/s. and **x = 5** km/s.

Problem – 5.

At 9 AM, a high-speed train departs from point A heading towards point C. At the same time, two passenger trains, one heading towards point A and the other towards point C, depart from point B, located between points A and C. The speeds of the two passenger trains are equal. After the high-speed train departs, within no more than 3 hours, it meets the first passenger train, then passes point B no earlier than 2 PM. Finally, 12 hours after meeting the first train, the high-speed train arrives at point C at the same time as the second passenger train. Find the time it takes for the first passenger train to reach point A.

Solution:

Let the speed of the high-speed train be **x**, the speed of the passenger trains be **y**, and the distance between A and B be **S**. The time taken for the high-speed train to meet the first passenger train is no more than 3 hours:

$$\frac{S}{x+y} \leq 3. \qquad (1)$$

The time taken by the high-speed train to pass point B is no less than 5 hours.

$$\frac{S}{x} \geq \mathbf{5}. \qquad (2)$$

The time for the high-speed train to catch up with the second passenger train is equal to $\frac{S}{x+y} + 12$, so the following equation holds:

$$\left(\frac{S}{x+y} + 12\right)(x - y) = S. \qquad (3)$$

We need to use inequalities (1) and (2) and equation (3) to find $t = \frac{S}{y}$

From this, we get $S = ty$. Substituting this expression for **S** into the above equation and inequalities, and letting $u = \frac{x}{y}$, we get the system:

$$t \leq 3(u + 1) \qquad (4)$$

$$t \geq 5u \qquad (5)$$

$$t = 6(u^2\text{-}1) \qquad (6)$$

By substituting ttt from equation (6) into inequalities (4) and (5), we obtain the system of inequalities:

$$2u^2 - u - 3 \leq 0 \quad \text{and} \quad 6u^2 - 5u - 6 \geq 0$$

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By solving the system of inequalities together and considering that u is positive, we obtain the unique solution $u = \frac{3}{2}$. Finally, substituting this solution into equation (6), we get: $t = \frac{15}{2} = 7{,}5$

Thus, the first passenger train will reach point A 7.5 hours after it departs, which means it will arrive at 16:30 (4:30 PM).

Problem 6. Two salt solutions are given. To create a new solution containing 10g of salt and 90g of water, it is required to take twice as much of the first solution compared to the second one. After one week, it is known that 200g of water evaporates from each kilogram of the first and second solutions. After this, to obtain the new solution in the same proportion, four times as much of the first solution should be taken compared to the second one. Determine how much salt is in 100 grams of each solution initially.

Problem 7.

There are two metal alloys with different percentages of copper, with weights of m kg and n kg respectively. A piece is cut from each alloy, and when the cut pieces are mixed with the remaining portions of the alloys, the copper percentage in both resulting alloys is exactly the same. Find the weight of the pieces that were cut.

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