PRICE AND SALES VOLUME FORECASTING USING QUANTUM SUPPORT VECTOR REGRESSION

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Abstract: This article analyzes the issue of accurately predicting the price and sales volume of retail products based on classical and quantum approaches. The classical Support Vector Regression (SVR) model was evaluated in comparison with the Quantum Support Vector Regression (QSVR) model based on quantum computing. Latent representations of the data were obtained through dimensionality reduction using an autoencoder and utilized for forecasting. The experimental results demonstrated that the QSVR model outperforms the classical SVR model in terms of MAE, MSE, and R2 indicators. The study effectively employed quantum kernel functions, data re-uploading feature maps, and the COBYLA optimization algorithm. The findings confirm the practical potential of quantum machine learning methods.

Introduction

Machine learning methods are currently widely used in various fields, particularly in applications for reliable and accurate price forecasting. This approach is especially important for dealership companies engaged in retail trade, as it enables them to develop optimal pricing and resale strategies by determining the current and future market value of their inventory or products.

In recent years, various studies have been conducted on assessing the residual value of retail goods or forecasting their prices based on ML, and these methods have been successfully tested in practice. It has been proven that machine learning and automated machine learning methods have demonstrated high performance on various types of sales data.

Although the above methods provide fairly accurate results, they usually require a large amount of computational resources. Quantum computers and the associated concept of quantum supremacy play a special role in solving problems with high computational power requirements. It is a new direction based on quantum technologies - quantum machine learning - that is laying the foundation for more efficient, highly accurate, and resource-saving ML models.

One of the widely used methods in classification and regression problems is the support vector machine (SVM), which is based on kernel matrices. Although the choice of kernel function depends on the characteristics of the problem, QML allows for the creation of powerful quantum kernels in this regard.

In this study, we will attempt to solve the problem of price and sales volume forecasting in retail using advanced approaches such as fidelity and projected quantum kernels. During the analysis, previously recommended quantum kernels are evaluated, and their results are compared with classical SVM kernels.

Our dataset also includes categorical features. To utilize them, we use one-hot encoding. This, in turn, significantly increases the data dimensionality and limits the capabilities of quantum devices. To address this issue, we employ an autoencoder approach, thereby reducing dimensionality while preserving the necessary information. At the conclusion of the study, the effectiveness of (Q) SVM methods is analyzed, and the impact of the autoencoder approach on both encoded and non-encoded data is examined.

2. Relevant scientific works

2.1 Machine learning in price and sales volume forecasting

Numerous studies aimed at determining price and sales volumes show the effectiveness of machine learning methods in high-precision forecasting. In 2017, Zong estimated the residual value of articulated trucks using multiple regression models. In 2018, Chiteri conducted an analysis based on auction and resale data on trucks.

In 2021, Milosevic developed an approach based on ensemble regression models to predict the cost of more than 500,000 pieces of construction equipment placed in the US market. Similarly, in 2021, Shehadeh and Alshboul determined the residual value of six types of construction equipment using various regression methods based on open auction data.

In 2023, Stühler et al. compared seven advanced ML models and three AutoML approaches for 10 different Caterpillar techniques based on 2,910 records obtained from a real online trading platform and evaluated the resulting effectiveness.

These studies confirm the relevance and practical effectiveness of machine learning in price and sales forecasting.

2.2 Quantum Machine Learning

Combining machine learning with quantum computing is becoming increasingly relevant in order to achieve an advantage over classical methods. It is the parallel computing capabilities of quantum computers that create wide opportunities for accelerating and simplifying machine learning tasks.

Quantum machine learning (QML) is a field specializing in the application of quantum algorithms to classical ML problems, and approaches developed on the basis of support vector machines (SVM) play an important role in this area.

Although significant advantages in QML have been proven mainly on the basis of algorithms implemented on error-resistant quantum computers, application programs still remain within the framework of medium-scale quantum computers.

In this article, we chose the quantum support vector machines (QSVM) approach due to the following key factors:

— It has been theoretically proven that QSVMs work exponentially faster than classical algorithms in some computational problems.

— SVM models are deeply studied from a mathematical point of view, for which accuracy, stability, convergence, and error limits are precisely assessed.

— QSVMs are based on surface quantum circuits suitable for medium-scale quantum computing systems, which makes them more practical.

The aim of the research is to increase the accuracy of predictions using quantum nuclear techniques and to empirically assess their advantages over classical models.

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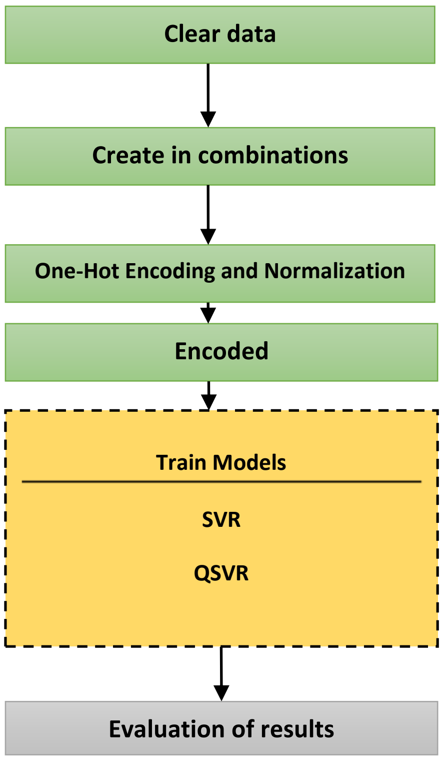

Figure 1. Stages of data preparation and modeling for the experiment

3. Metodology

In order to study the effectiveness of quantum machine learning approaches, in this study, quantum support vector regression (QSVR) models are evaluated in comparison with the classical support vector regression (SVR) model.

A general overview of the methodological process is presented in Figure 1, which includes all stages from data preparation to model testing and final evaluation.

3.1 Data Set Creation

Preliminary data were collected regularly over 15 months by collecting sales data for 51 major retail products. Table 1 presents the characteristics collected and selected for analysis.

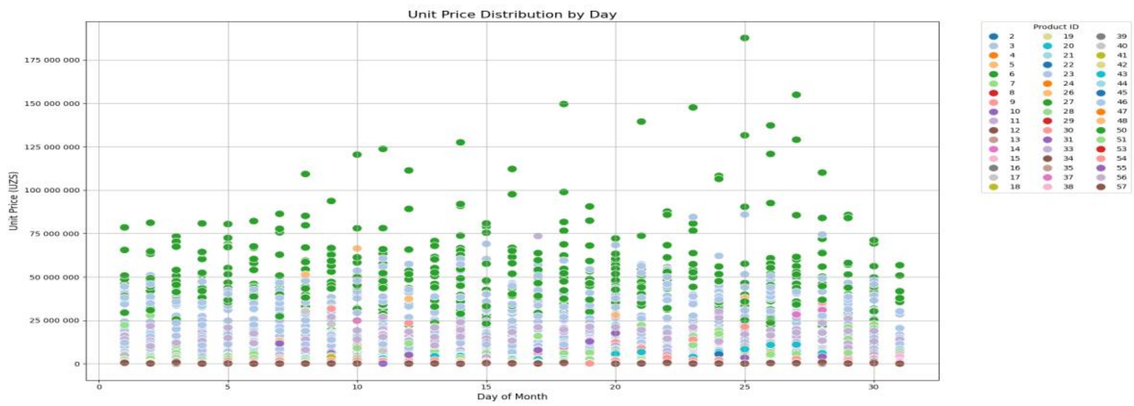

Figure 2. Graph of the relationship between sales time (x-axis) and price (y-axis) in a set of product sales data.

Figure 2 shows the relationship between product sales time and price.

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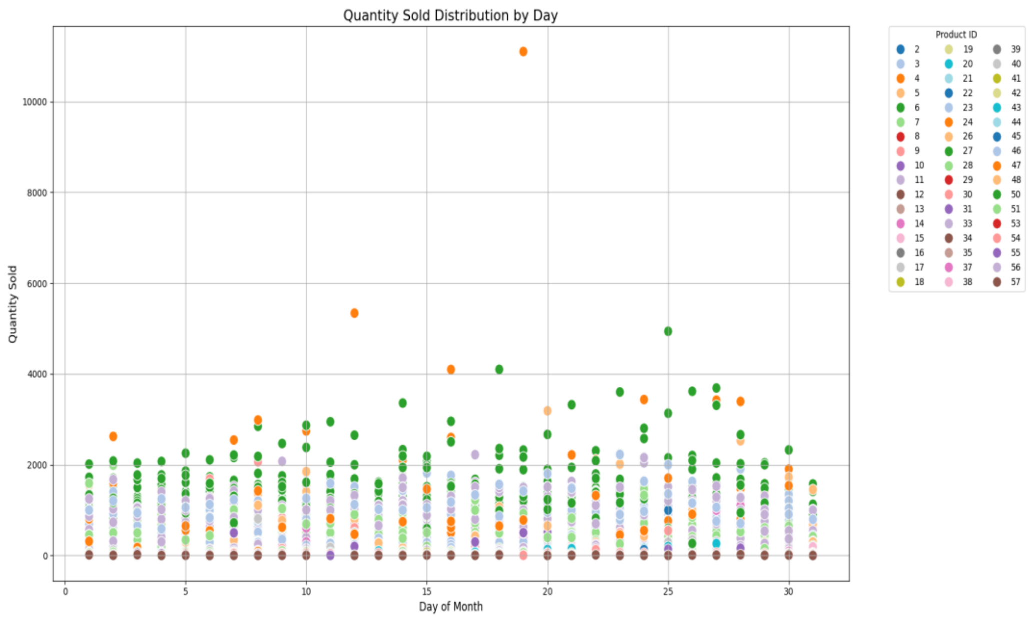

Figure 3. Graph of the relationship between sales time (x-axis) and quantity (y-axis) in a set of product sales data.

This Figure 3 shows the relationship between product sales times and sales volume.

Table 1. Characteristics in collected datasets, their types and typical values

Property name	Type	Sample
Product code (product_id)	Digital	6
Product category	Categorical	SIMPLE
Sale date (date)	Time	2024-07-18 16:21:05
Agent region (agent_id)	Digital	38
Product selling price (unit_price)	Digital	25800
Product sales amount (quantity_sold)	Digital	30
Holiday	Categorical	YES/NO

3.2 Data Clearing and Preparation

Duplicate records were identified by iterative comparison of a combination of different properties and removed from the dataset. The amount of sales and price values (outlier) were normalized using the Min-Max normalization method using formula (1).

$$x_{new} = \frac{x - x_{\min}}{x_{\max} - x_{\min}} \quad (1)$$

The processing of missing values was carried out depending on the type of attributes. Also, the dataset was enriched by extracting calendar properties from the date attribute. More precisely, such columns as day (day), month (month), year (year), and week number (week) were separated from the sales date, which made it possible to take into account seasonality and changes occurring over time in the model.

Based on these new columns, the set of properties was improved and brought to the state of the main set, that is, a structure was formed that covers the most important attributes related to time and category. This approach served to increase the accuracy of forecasting by making each model sensitive to time components. Table 2 shows a fragment of the data structure formed on the basis of these changes.

Table 2. Updated attribute structure based on sales data

day	month	year	product_id	agent_id	category	week_name	holiday	unit_price	quantity_sold
0.9666	0.5454	0.5	0.3454	0.7733	0	0.1667	0	0.0038	0.0039
0.9666	0.2727	1	0.8	1	0	0.3333	0	0.0884	0.0883
0.1333	0.0909	1	0.4363	0.6	0	0.3333	0	0.0001	0.0019
0.7	1	0.5	0.0909	0.5333	0	1	0	0.0182	0.0221
0.0666	0.7272	0.5	0.4363	0.5733	0	0.1666	0	0.0002	0.0080
0.6666	0	1	0.4363	0.6	0	0.1666	0	0.0001	0.0019
0.5666	0.5454	0.5	0.0909	0.5333	0	0.5	0	0.0023	0.0029
0.1667	0.6363	0.5	0.0909	0.6533	0	0.1666	0	0.0020	0.0015
0.7333	0	0.5	0.2909	0.5333	0	0.1666	0	0.0070	0.0059
0.5	0	1	0.5272	0.6	0	0.5	0	0.0017	0.0015

3.2 Auto encoder-based dimensional compression and controlled prediction

In this study, the Autoencoder architecture was adapted not only for data size compression, but also for predicting the target variable based on controlled learning. Unlike traditional Autoencoders, in this approach, instead of a decoder layer, a regression module is placed as an output. As a result, the latent representations created by the encoder were directly transferred to the prediction model. (Fig. 4)

The general structure of the autoencoder model is as follows:

Encoder: projects input attributes into a compressed, low-dimensional latent space;

Latent layer: represents compressed, information-dense representation;

Regressor: A layer that projects data from a latent space onto a target variable.

The attributes used in the model consist of several combinations, each of which constantly contains such basic properties as "quantity_sold" and "unit_price." These attributes are defined as the main set, and various combinations are formed by adding other parameters (for example, holiday, agent_id, week_name, category).

In the process of processing data combinations, 10 was defined as the maximum size of the latent space. Accordingly, the following rules were applied:

If the number of input attributes is greater than 10, Autoencoder compresses them into a 10-dimensional latent space;

If the number of input attributes is 10 or less, the latent space size is set equal to the input size.

With this approach, comparable and consistent compressed representations were created for each combination of attributes. These representations in the latent space were transmitted as input data for subsequent regression models.

Training of the autoencoder model was carried out on the basis of the following technical configurations:

— **Optimizer algorithm:** Adam
— **Loss function:** Mean Squared Error (MSE)
— **Encoder activation:** ReLU
— **Regressor activation:** Linear

The regression problem based on compressed latent values was solved in two different models:

1. Quantum approach - implemented using the Quantum Variational Regressor (QVR) model developed on the Qiskit platform. The COBYLA algorithm was used to optimize the model parameters. The latent vectors obtained as a result of the autoencoder were transmitted encoded to the quantum period, and the regression results were obtained based on this quantum architecture.

2. Classical approach - a Support Vector Regression (SVR) model was built based on compressed data. This model served to effectively predict the high-dimensional input space through nuclear projections.

Thanks to this approach, Autoencoder served not only as a means of reducing dimensionality, but also to improve the overall performance of quantum and classical prediction models by creating semantically meaningful latent representations.

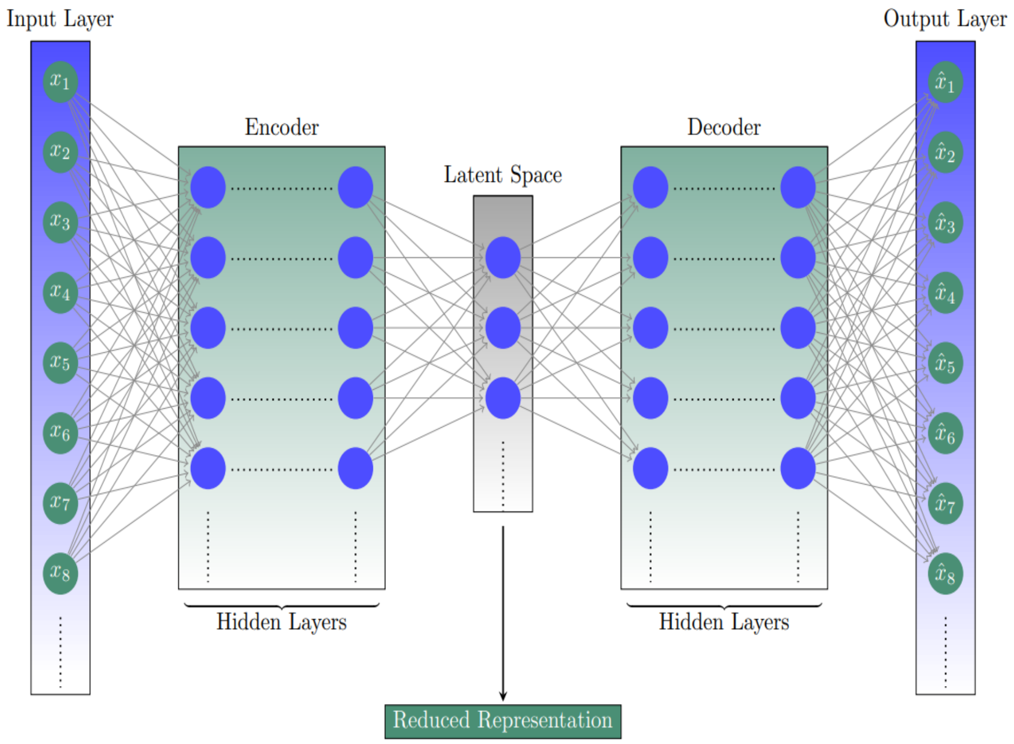

Figure 4. A typical autoencoder consists of two deep neural networks, each consisting of several dense layers.

3.3 Feature Maps

In quantum machine learning, feature maps play an important role in encoding classical data into quantum states and converting them into quantum cores. In this case, each data point is encoded by a special quantum circuit, and as a result, it becomes possible to effectively study spatial, nonlinear connections.

The data re-uploading method is particularly effective in QML algorithms. In this approach, incoming data is encoded several times sequentially into a quantum circuit, and after each step, qubits are connected to each other using CNOT doors. Such multi-stage coding gives the model high expressiveness and helps to effectively study hidden patterns in complex data.

Figure 6 shows a quantum feature map diagram based on this data re-uploading. Here, in each qubit, data is encoded multiple times using RY rotation doors, and each time qubits are interconnected with CNOT doors. As a result of this approach, the expressiveness of the quantum kernel increases, and the possibility of constructing predictive models with higher accuracy compared to classical kernel methods is created.

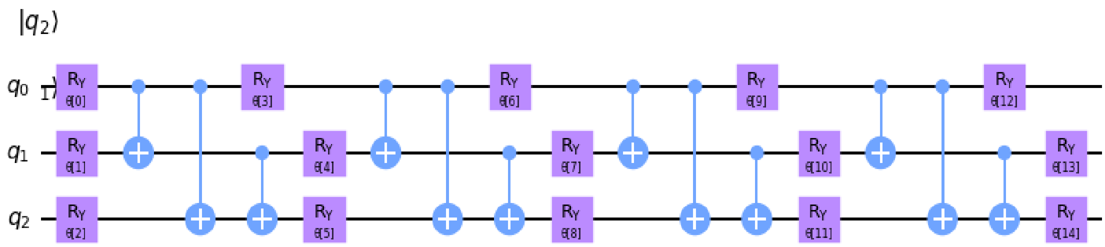

Figure 6. Quantum property map based on data re-uploading

In this scheme, data in each qubit is encoded multiple times through RY doors, and between each encoding step, qubits are connected using CNOT doors. Such an architecture increases the spatial expressiveness of the model and allows for the study of complex nonlinear relationships.

3.4 Quantum Support Vector Regression (QSVR)

Quantum support vector regression (QSVR) is an improved version of the traditional support vector regression (SVR) model based on quantum computation, which allows ensuring high efficiency in complex regression problems. This approach utilizes the advantages of quantum computers in high parallelism and dimensional data processing. Especially when modeling the relationships between nonlinear properties, it allows optimal placement of the hyperplane in high-dimensional space by representing quantum states as vectors. The QSVR model adapts them to calculation using quantum algorithms while preserving the basic concepts of the classical SVR model (core function, extreme width, loss function).

As in the traditional SVR model, the goal in this approach is to construct an optimal regression function that predicts the output values of y_i in accordance with the given input vectors $X = \{x_i\}_{i=1}^{n}$. The mathematical basis of the QSVR model is as follows. First, the data is projected into a high-dimensional quantum property space through the mapping function $\phi(x)$. As a result, the nonlinear regression problem is converted to a linear one.

The QSVR model is defined by the following linear estimation function:

$$f(x) = \langle w, \phi(x) \rangle + b \qquad (7)$$

here w — gravity vector, b — fixed parameter, $\phi(x)$ — property mapping function in quantum space.

The main goal of the model is to find parameters w and b such that they provide minimal losses based on the sensitive loss function ε.

The sensitive loss function - ε used in the QSVR model is expressed as follows:

$$L_\varepsilon(y_i, f(x_i)) = \begin{cases} 0 & agar\,|y_i - f(x_i)| \le \varepsilon \\ |y_i - f(x_i)| - \varepsilon, & aks\,holda \end{cases} \quad (8)$$

The optimization function of the model, along with minimizing the squared loss, serves to maintain the generalizability of the model:

$$\min_{w,b,\xi_i,\xi_i^*} \left(\frac{1}{2}\|w\|^2 + C\sum_{i=1}^{n}(\xi_i + \xi_i^*)\right) \quad (9)$$

Conditions:

$$\begin{cases} y_i - \langle w, \phi(x_i)\rangle - b \le \varepsilon + \xi_i \\ \langle w, \phi(x_i)\rangle + b - y_i \le \varepsilon + \xi_i^* \\ \xi_i, \xi_i^* \ge 0 \end{cases} \quad (10)$$

where C is the adjustment constant, which regulates the balance between the complexity and accuracy of the model, and ξ_i, ξ_i^* represents errors located outside the tolerance threshold.
The main difference of the QSVR model is that the kernel function is calculated using a quantum algorithm. In this case, the classical kernel function:

$$K(x_i, x_j) = \langle \phi(x_i), \phi(x_j)\rangle \quad (11)$$

instead, the amplitude between the quantum states is taken as the inner product:

$$K_Q(x_i, x_j) = |\langle \psi(x_i), \psi(x_j)\rangle|^2 \quad (6)$$

where $\psi(x_i)$ is the input vector encoded as a quantum state. The quantum nucleus $K_Q(x_i, x_j)$ is calculated using special quantum circuits. Often these nuclei are evaluated using the Quantum Kernel Estimation (QKE) or Swap-Test algorithm.
To solve the model, the following dual optimization function is generated:

$$\max_{a_i, a_i^*}\left(-\frac{1}{2}\sum_{i,j=1}^{n}(a_i - a_i^*)(a_j - a_j^*)K_Q(x_i, x_j) + \sum_{i=1}^{n} y_i(a_i - a_i^*) - \varepsilon\sum_{i=1}^{n}(a_i + a_i^*)\right) \quad (7)$$

Conditions:

$$\sum_{i=1}^{n}(a_i + a_i^*) = 0, \quad 0 \le a_i, a_i^* \le C \quad (8)$$

At the end of the model, the decision function is expressed as follows:

$$f(x) = \sum_{i=1}^{n}(a_i - a_i^*)K_Q(x_i, x) + b \quad (9)$$

where only vectors $a_i - a_i^* \ne 0$ are support vectors.
Advantages of the QSVR model:

- Effective operation in large-dimensional spaces;
- Optimal modeling of nonlinear relations by calculating the quantum kernel;
- The possibility of high-precision forecasting with small samples.

In practice, QSVR models can be created and simulated using quantum computing libraries such as Qiskit, PennyLane, or Cirq. This method can be used especially in real-time regression problems, economic forecasting, environmental monitoring, and scientific computing.

3.5 Performance Metric

The RMSE assessment indicator is one of the widely used indicators for assessing the accuracy of regression models. It mainly determines the difference between the values of the target variable obtained using forecasts in a set of economically significant data. The form of the RMSE equation is as follows.

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(Y_i - Y_i)^2}$$

In this formula, n is the number of data in the dataset, Y_i is the variable representing the true value of the target variable, represents the true value of the i-th row, and Y_i represents the predicted values of the target variable.

The MAE evaluation indicator is a convenient indicator for assessing the accuracy of machine learning algorithms. This indicator determines the absolute difference between the forecasted main values and the actual initial values of the target variable in the dataset. The evaluation indicator MAE is expressed by the following formula:

$$MAE = \frac{1}{n}\sum_{i=1}^{n}\left|Y_i - Y_i\right|$$

The evaluation indicator MAE is calculated by calculating the absolute value of the difference between the projected and actual given values and averaging them. This method mainly calculates the average magnitude of sharp or low-grade errors without considering the direction of errors.

The evaluation indicator R^2 is also widely known as the coefficient of determination, the evaluation indicator R^2 is an effective indicator for determining the effectiveness of regression models. The method mainly allows measuring the stability of the algorithm by comparing the variability of predicted values with the dynamic variability of the initial actual values of the target variable. The formula for the R-square estimate indicator is as follows:

$$R^2 = 1 - \frac{\sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2}{\sum_{i=1}^{n}(Y_i - \tilde{Y}_i)^2}$$

4. RESULTS

In this study, the effectiveness of support vector regression models (SVR and QSVR) built on the basis of classical and quantum approaches was analyzed. All experiments were conducted in the Python environment using Qiskit (IBM-Qiskit) quantum computing libraries. To ensure the reliability of the results, the data set was checked according to a hold-out validation scheme, divided into 80% training and 20% test sets.

Table 3 presents the main evaluation criteria for the SVR and QSVR models - mean absolute error (MAE), mean square error (MSE), and coefficient of determination (R^2). The results show that the QSVR model showed an advantage over the classical SVR in all key metrics.

Table 3. Prediction results for the SVR and QSVR models (according to the test set).

Model	MAE	MSE	R^2 Score
SVR	0.120	0.410	0.674
QSVR	0.077	0.069	0.780

As can be seen from the table above, in the QSVR model, the values of MAE and MSE are low, and the R2 indicator is high, i.e., the accuracy of the quantum model in information prediction is higher compared to the classical model.

Figure 7 and Figure 8 below show a comparative graph of the actual and predicted values in the test set for the SVR and QSVR models. These graphs show the discrepancies between the predicted results and the actual results, as well as how accurate the model results are.

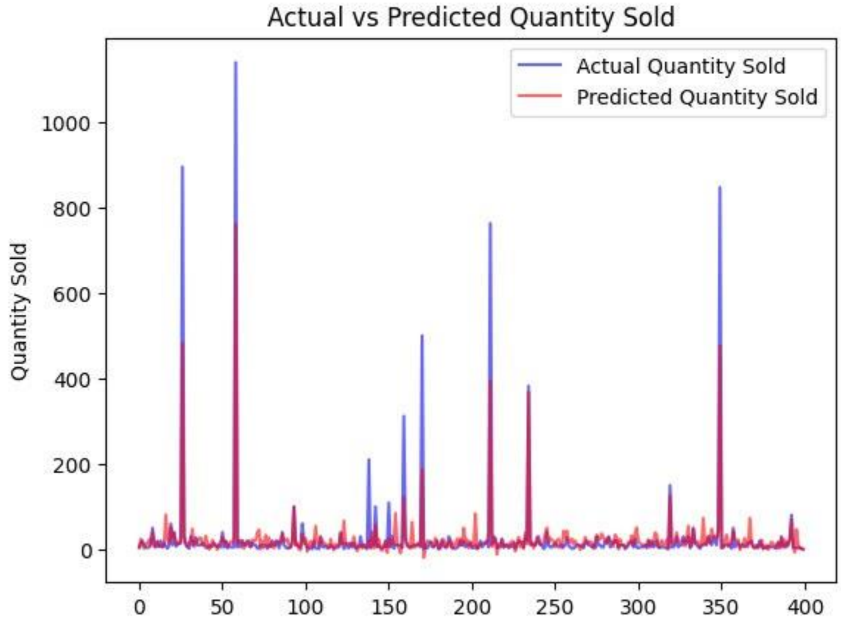

Figure 7. Diagram of the distribution of actual and predicted values for the SVR model.

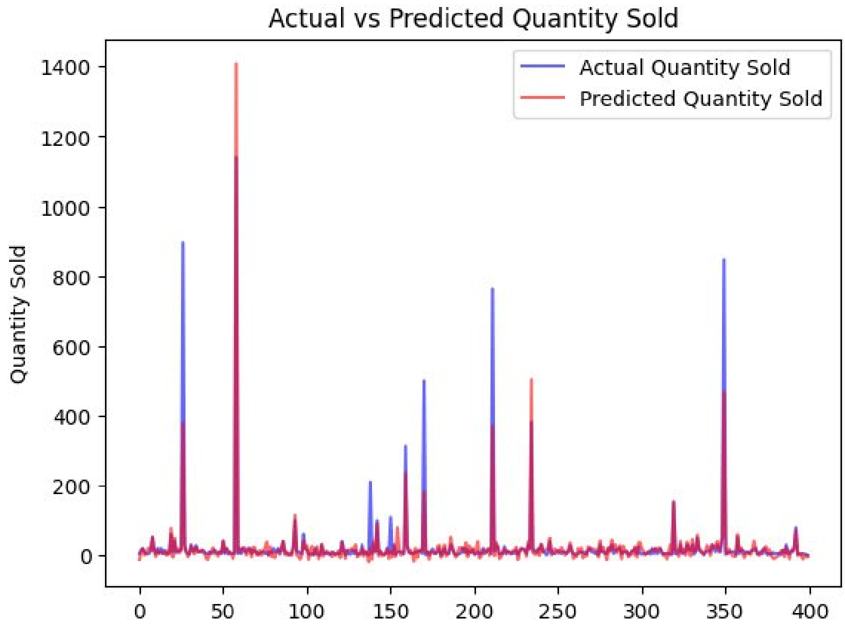

Figure 8. Diagram of the distribution of actual and predicted values for the QSVR model.

5. CONCLUSION

In this study, the problem of forecasting the price and sales volume of retail products was solved using the methods of classical and quantum support vector regression (SVR and QSVR). In the conducted experiments, the effectiveness of both models was assessed using key statistical indicators such as RMSE, MAE, and R^2. The results proved that the quantum support vector regression (QBR) method has higher efficiency compared to the classical SVR.

In particular, the QSVR model showed significantly lower values for MAE and MSE indicators compared to the SVR model, and a higher result for the R^2 indicator.

Also, the Autoencoder approach was used in the study to reduce the size of high-dimensional data. The representations in the latent space created as a result of this approach served as an effective and informative introduction to regression models. With the help of the autoencoder method, the overall predictive effectiveness of quantum and classical regression models has been improved, and high results have been achieved, especially when used in conjunction with quantum regression.

The research results confirmed the possibilities of quantum machine learning methods for working with high accuracy and efficiency in small and medium-sized datasets. QSVR models constructed using the data re-uploading method showed particularly high results in determining nonlinear and complex relationships.

From a practical point of view, the use of quantum algorithms can be very suitable for real-time decision-making and working with small samples, reducing the need for computational resources. In particular, the implementation of quantum technologies in such areas as retail trade, economic forecasting, and environmental monitoring is of great practical importance.
In the future, it is possible to test the results of this research on more complex and complex data sets, further improve quantum algorithms, and deepen the practical possibilities of quantum machine learning approaches by conducting experiments on a real quantum apparatus

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