



MATHEMATICAL MODELS OF INDUCTION TRANSDUCERS FOR VIBRATION MEASUREMENT

Amirov S.F.
Shoimov Y.Y.
Sharapov Sh.A.
Tog'ayev A.S.

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Abstract.

It is well known that the metrological characteristics of induction vibration transducers (IVTs) are largely dependent on the condition of their magnetic fields. In particular, the main technical (static and dynamic) characteristics of the developed inertial element (IVTs) depend on analytical expressions that describe the relationship between the magnetic flux in the working air gaps and the coordinates of the inertial element. In other words, it is extremely important that the mathematical models of the magnetic circuits of IVTs accurately reflect the electromagnetic processes occurring in the magnetic systems of these transducers.

Keywords: Induction vibration transducer, inertial element, energy-information model, magnetic conductors, magnetic circuit, reactive impedance.

Induction vibration transducers, including the developed IVTs, possess certain specific features in their magnetic circuits. For example, in IVTs, the working magnetic flux is typically generated by permanent magnets or electromagnets with excitation windings connected to a DC power source [2]. As a result, in this class of IVTs, the magnetic flux is modulated by vibrational oscillations—that is, pulsating magnetic fluxes are generated in the magnetic circuits of IVTs with frequencies equal to the frequencies of the vibrations. Consequently, eddy currents are generated in the ferromagnetic parts of the IVTs' magnetic circuits [8]. This phenomenon becomes especially significant when the frequency of the input vibration oscillations is very high, necessitating the consideration of both active and reactive components of the magnetic circuit resistance during calculations.

Before developing mathematical models for the magnetic circuits of IVTs, the following constraints are introduced, which are applicable to this type of magnetic circuit [7]:

1. Each sector of the inertial element (IE) made of magnetic conductive material has identical geometric dimensions and is made of magnetically soft (serving as magnetic conductors) and magnetically hard (serving as permanent magnets) materials with identical magnetic properties;
2. The quarter segments of the ring-shaped external magnetic conductor, divided by horizontal and vertical axes, have identical geometric dimensions and are made of magnetically soft material with uniform magnetic properties;
3. The four pairs of pole tips in the magnetic circuit and the arc-shaped working air gaps between them have identical geometric dimensions and are made of magnetically soft material with uniform magnetic properties.
4. Since the ratio $(\delta_{\text{нш}}/R_{\text{yp}}) \ll 1$ holds true for the corresponding ring-shaped working air gaps of permanent magnets, the magnetic field lines in these air gaps are mutually parallel;

5. The magnetically soft materials in the magnetic circuit operate in the linear portion of the primary magnetization curve, meaning that the magnetic circuit belongs to the class of linear circuits (i.e., magnetic properties of the soft magnetic materials do not depend on the value of the magnetic field induction within them);

6. The magnetic fluxes that merge through the side surfaces and edges of the magnetic conductors are considered to be zero, that is, their values are negligible compared to the working magnetic fluxes and are therefore not taken into account. These constraints do not significantly reduce the calculation accuracy, but they simplify the computations considerably.

It is worth noting that in magnetic circuit analysis, equivalent circuit diagrams that are analogous to electric circuits are commonly used. In this classical analogy, the magnetomotive force (MMF) in the magnetic circuit corresponds to the electromotive force (EMF) in an electric circuit; the drop in magnetic potential along a part of the magnetic circuit is analogous to the voltage drop across a component in an electric circuit; the magnetic flux corresponds to electric current; the magnetic reluctance of a part of the magnetic circuit corresponds to electrical resistance; and the magnetic reactance arising from eddy currents due to alternating magnetic flux is analogous to the reactive impedance in an electric circuit.

However, this classical analogy has certain drawbacks. For example, in electric circuits, the product of voltage and current is measured as power, but this criterion does not apply in magnetic circuits. Additionally, when current flows through resistance in electric circuits, heat is generated—this property does not exist in magnetic circuits, and so on.

To eliminate the above and other shortcomings of the classical analogy, Professor M.F. Zaripov developed an energy-information model that is equally applicable to circuits of various physical natures (electrical, mechanical, magnetic, hydraulic, etc.) [8].

According to this, the magnetic flux in the magnetic circuit ($\Phi = Q_\mu$) corresponds to the electric charge (Q_3) in the electrical circuit; the time derivative of the magnetic flux ($\frac{d\Phi}{dt} = \frac{dQ_\mu}{dt} = I_\mu$) corresponds to the electric current ($\frac{dQ_3}{dt} = I_3$); the active conductivity of the magnetic material due to eddy currents generated by the alternating magnetic flux in the magnetic circuit ($R_\mu = G_{3, \text{yюp.}}$) corresponds to the active resistance (R_3) in the electric circuit; the electrical capacitance of the magnetic material in the path of the eddy currents in the magnetic circuit ($L_\mu = C_{3, \text{yюp.}}$) corresponds to the inductance (L_3) of the electrical circuit; and finally, in classical analogy, the magnetic reluctance of a part of the magnetic circuit is considered analogous to the capacitance in an electrical circuit. According to this analogy, all the properties that apply to an electrical circuit (referred to as criteria in this new analogy) also apply to other circuits of different physical nature, including the magnetic circuit.

The required geometric dimensions of a quarter part of the IVU magnetic circuit and its structural schematic are shown in Figure 1.,

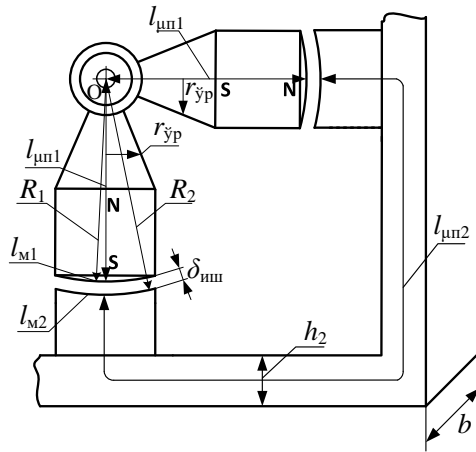


Figure 1. Structural schematic of the magnetic circuit of the IVU (Induction Vibration Transducer)

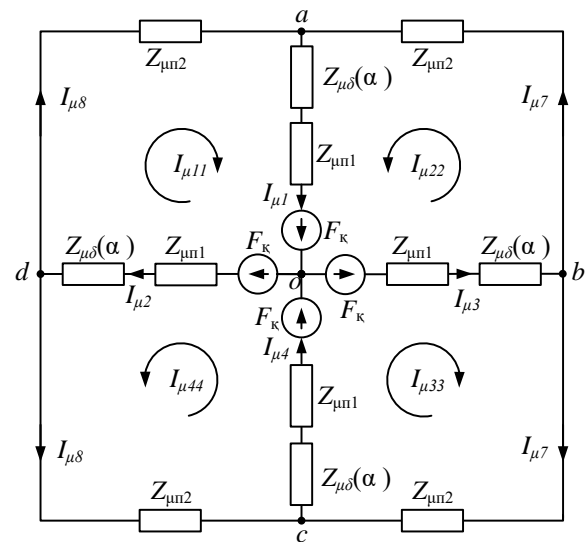


Figure 2. Equivalent schematic of the magnetic circuit of the IVU (Induction Vibration Transducer)

"The equivalent circuit constructed taking into account the above-mentioned constraints and the energy-information model is shown in Figure 2.

For this magnetic circuit, the system of equations constructed based on the loop current method is as follows [21, pp. 111-123]."

$$\begin{cases} Z_{\mu 11}I_{\mu 11} + Z_{\mu 12}I_{\mu 22} + 0 \cdot I_{\mu 44} + Z_{\mu 14}I_{\mu 44} = 2F_K, \\ Z_{\mu 21}I_{\mu 11} + Z_{\mu 22}I_{\mu 22} + Z_{\mu 23}I_{\mu 33} + 0 \cdot I_{\mu 44} = 2F_K, \\ 0 \cdot I_{\mu 31} + Z_{\mu 32}I_{\mu 22} + Z_{\mu 33}I_{\mu 33} + Z_{\mu 34}I_{\mu 44} = 2F_K, \\ Z_{\mu 41}I_{\mu 41} + 0 \cdot I_{\mu 42} + Z_{\mu 43}I_{\mu 33} + Z_{\mu 44}I_{\mu 44} = 2F_K, \end{cases} \quad (1)$$

here $Z_{\mu 11} = Z_{\mu 22} = Z_{\mu 33} = Z_{\mu 44} = 2Z_{\mu \delta}(\alpha) + 2Z_{\mu n1} + Z_{\mu n2}$ - the total complex magnetic resistance of the corresponding independent circuit in the magnetic circuit, $[H^{-1}]$; $Z_{\mu \delta}(\alpha) = \frac{W_{\mu \delta}(\alpha)}{\omega_M}$, $[\Omega^{-1}]$; $W_{\mu \delta}(\alpha) = \frac{\delta_{\mu n}}{\mu_0 b l_{yp}}$, $[H^{-1}]$ - the modulus value of magnetic resistance and magnetic hardness of the arc-shaped working air gap between pole machines (according to the energy-information model of chains of different physical nature, the parameter is inverse to the magnetic capacity);- $Z_{\mu n1} = R_{\mu n1} + j \left(\omega_M L_{\mu n1} - \frac{W_{\mu n1}}{\omega_M} \right)$, $[\Omega^{-1}]$; $R_{\mu n1} = G_{\mu n1}$, $[\Omega^{-1}]$; $L_{\mu n1} = C_{\mu n1}$, $[F]$; $W_{\mu n1} = \frac{l_{\mu n1}}{\mu \mu_0 \pi r_{yp}^2}$, $[H^{-1}]$ - Active magnetoresistance inductance and magnetic hardness of sector IE, respectively; $Z_{\mu n2} = R_{\mu n2} + j \left(\omega_M L_{\mu n2} - \frac{W_{\mu n2}}{\omega_M} \right)$; $R_{\mu n2}$, $[\Omega^{-1}]$; $L_{\mu n2}$, $[F]$; $W_{\mu n2} = \frac{l_{\mu n2}}{\mu \mu_0 b h_2}$, $[H^{-1}]$ - 0- Same parameters as above for quarter section of magnetic conductor; $Z_{\mu 12} = Z_{\mu 14} = Z_{\mu 21} = Z_{\mu 23} = Z_{\mu 32} = Z_{\mu 34} = Z_{\mu 41} = Z_{\mu 43} = Z_{\mu \delta}(\alpha) + Z_{\mu n1}$ - the total magnetic resistance between the respective independent circuits in the magnetic circuit; F_K - MYuK in the corresponding independent inter-circuit branch of the magnetic circuit; $\mu_0 = 4\pi \cdot 10^{-7}$ $[H/m]$ - permanent magnet.

It is worth noting that the magnetic stiffness ($W_{\mu\delta}(\alpha)$) of the air gap between coaxially arranged pole tips whose arcs form an angle $\alpha < 90^\circ$ measured between their radii at the arc ends, is usually determined using the following formula [10]:

$$W_{\mu\delta}(\alpha) = \frac{\ln(R_2/R_1)}{\mu_0 \alpha b}. \quad (2)$$

The conducted calculations show that..., $\frac{R_2-R_1}{R_{\text{yp}}} \ll 1$ when the condition is met $W_{\mu\delta}(\alpha) = \frac{\ln(R_2/R_1)}{\mu_0 \alpha b} \approx \frac{\delta_{\text{иш}}}{\mu_0 b l_{\text{yp}}}$, It can be considered that.

The system of equations (1) represents the mathematical model of the magnetic circuits of inductive vibration transducers (IVTs) with a wide measurement frequency range.

The analysis of the system of equations (1) and the values of the magnetic resistances within it demonstrates the validity of the following relationships:

$$I_{\mu 11} = I_{\mu 22} = I_{\mu 33} = I_{\mu 44}, \quad (3) \quad I_{\mu 1} = I_{\mu 2} = I_{\mu 3} = I_{\mu 4} = I_{\mu \text{иш}}, \quad (4)$$

$$I_{\mu 5} = I_{\mu 6} = I_{\mu 7} = I_{\mu 8} = I_{\mu \text{т}}, \quad (5) \quad I_{\mu \text{иш}} = 2I_{\mu \text{т}}. \quad (6)$$

Taking into account equations (3) to (6), it is sufficient to calculate the magnetic current in one independent loop of the equivalent circuit shown in Figure 1, that is:

$$I_{\mu \text{иш}} = \frac{4F_K}{4Z_{\mu\delta}(\alpha) + 4Z_{\mu\pi 1} + Z_{\mu\pi 2}}. \quad (7)$$

It should be noted that the created inductive vibration transducers (IVTs) are used in the control, management, and diagnostic systems of railway transport objects, where the vibration frequency range typically falls within 2–12 Hz [5]. Therefore, it becomes possible to adopt an additional simplification — namely, to neglect energy losses due to eddy currents — by utilizing amorphous magnetically soft materials (such as AMAG 492 alloy [6]) in which such losses in alternating magnetic fields are extremely small.

In this case, the magnetic circuit will consist only of the W_μ parameters, and magnetic currents (I_μ) can be replaced with magnetic fluxes (Q_μ).

If the aforementioned additional simplification is taken into account, then equation (7) can be rewritten in the following form:

$$Q_{\mu \text{иш}} = 4F_K / 4W_{\mu\delta}(\alpha) + 4W_{\mu\pi 1} + W_{\mu\pi 2}. \quad (8)$$

By substituting the above-given value of $W_{\mu\delta}(\alpha)$ into expression (8), and considering that

$S_{\text{фол}} = b l_{\text{yp}} = \pi r^2 - \frac{\pi r R_{\text{yp}}}{180^\circ} \alpha$, $\alpha = \alpha_m \sin \omega_m t$ we obtain the following expression:

$$Q_{\mu \text{иш}} = \frac{4F_K \mu_0 \left(\pi r^2 - \frac{\pi \alpha_m r R_{\text{yp}}}{180^\circ} \sin \omega_m t \right)}{4\delta_{\text{иш}} + \mu_0 (4W_{\mu\pi 1} + W_{\mu\pi 2}) \left(\pi r^2 - \frac{\pi \alpha_m r R_{\text{yp}}}{180^\circ} \sin \omega_m t \right)}. \quad (9)$$

If the magnetic permeability of the material is assumed to approach infinity ($\mu \rightarrow \infty$), that is, if the magnetic reluctance of the ferromagnetic conductor is neglected in the calculations, then expression (9) takes the following simplified form suitable for practical use:

$$Q_{\mu \text{иш}} = \mu_0 \frac{F_K \left(\pi r^2 - \frac{\pi \alpha_m r R_{\text{yp}}}{180^\circ} \sin \omega_m t \right)}{\delta_{\text{иш}}} = Q_{\mu \text{иш.м}} - Q_{\mu \text{иш.мах}} \sin \omega_m t, \quad (10)$$

here $Q_{\mu \text{иш.м}} = \mu_0 F_K \frac{\pi r^2}{\delta_{\text{иш}}}$; $Q_{\mu \text{иш.мах}} = \mu_0 F_K \frac{\pi \alpha_m r R_{\text{yp}}}{180^\circ \delta_{\text{иш}}}$ - The steady (DC) component and the amplitude of the harmonic (AC) component of the working magnetic flux, respectively.

Nonlinear Regimes in Magnetic Circuits of Inductive Vibration Transducers (IVTs)

The magnetic circuits of inductive vibration transducers (IVTs) generally operate in the linear region of the primary magnetization curve. However, due to production requirements, in some cases, the excitation magnetic field intensity in IVTs must be increased to enhance their sensitivity. As a result, although the sensitivity of IVTs increases, their magnetic circuits move into the nonlinear, i.e., saturation region of the magnetization curve, causing the magnetic circuits to function in a nonlinear regime. Naturally, this negatively affects the static and dynamic characteristics of IVTs as well as their measurement accuracy [1]. In order to assess the magnitude of such negative effects, it becomes necessary to study the nonlinear regime using the above-mentioned example of the magnetic circuit.

Conclusion

Constructing the inertial element in the form of two diametrically opposed magnetic conductors with different masses and positioning it between horizontal and vertical axes allows for the measurement of both linear and torsional vibrations with high sensitivity in various directions.

The analysis of mathematical models that determine the change in the position of the inertial element's pole tips under the influence of acceleration applied to the transducer confirms that the active surface of the pole tips is linearly related to the coordinate of the inertial element.

The equilibrium equations of the electromagnetic mechanical forces, inertial, gravitational, and frictional forces acting on the inertial element were derived, enabling the theoretical investigation of the main characteristics of inductive vibration transducers.

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