NUMERICAL SOLUTION OF A ONE-DIMENSIONAL PROBLEM OF THE OIL FILTRATION PROCESS

Hoshimov Donobek
doctoral student of Tashkent University of Information Technologies named after Muhammad Al-Khwarizmi
https://doi.org/10.5281/zenodo.14499338

Abstract: In this scientific article, a one-dimensional view of the problem of numerical modeling of the oil filtration process, which is currently one of the most relevant topics, is numerically modeled and the results are analyzed.

Keywords: Numerical methods, filtration process, mathematical model, one-dimensional problem, debit.

Introduction

In the 21st century, with the increasing demand for global energy resources and the important role of the oil and gas industry in the economic growth of countries around the world, in-depth study of oil refining and filtration processes is becoming an urgent issue. In countries rich in natural resources, such as Uzbekistan, the improvement of these processes allows achieving high efficiency and saving resources. At the same time, the specific features of filtration processes and their modeling are of particular importance in ensuring energy security and ecological balance. Therefore, the issue of oil filtration is an integral part of the economy. The preservation, conservation and exploration of natural resources have a significant impact on the economies of many countries. This article sheds light on a one-dimensional view of the issue of oil filtration and analyzes the results.

The development of scientific and technological progress is impossible without the development of scientific, methodological, theoretical and practical foundations for building information processing systems based on mathematical modeling (MM) and computational experiments (CT). These systems are a synthesis of the achievements of traditional scientific methods and new information technologies in information processing and presentation. Their intellectual core is the triad "model-algorithm-program". This triad is connected with the object, as a result of which the chain "object - model - algorithm - program - computational experience" is formed. As one of the intensively developing areas of modern applied mathematics and computer science, it is necessary to highlight MM and CT. These methods are being developed to analyze and control the activities of complex dynamic objects, processes, technical systems, etc. The main advantage of such an approach is that by replacing the studied object or processes with sufficiently suitable mathematical models and conducting CT on modern computing clusters, it becomes possible to determine the influence of external and internal factors, to identify new hidden patterns, including in hydrodynamics, technical systems, etc. The development of the concept of MM theory is one of the crucial problems for fundamental sciences.

This theory combines the processes of algorithmization, information collection, processing, accumulation and presentation from the point of view of the general system as a unified, whole system. As a result, it makes it possible to solve significant socio-economic

IBAST | Volume 4, Issue 12, December

IBAST ISSN: 2750-3402

problems. The process of conducting experiments with a model in order to obtain information for studying an object is called "modeling". In this case, the model serves as the object of study and, by analogy or in some other way, reflects the object through a model that is to a certain extent suitable. That is, the model is a physical or abstract system that reflects the object of study to a certain extent. The study of stationary and dynamic processes and objects of study in MM constitutes a complex problem, a theoretical basis. This basis includes natural (physical, hydrodynamic, mechanical, etc.) laws. Knowledge and consideration of these laws when constructing logical models allows for a more accurate description of the process and the object of study. To select the most important factors and parameters affecting hydrodynamic and technical systems and to optimally control them, it is necessary to conduct computational experiments on a computer using developed numerical algorithms.

When developing a mathematical model, applying natural laws to the object, we create an information model (AM), which includes complete information for the research process. The selection of the most important information when creating AM and its complexity depends on the goals of the MM. Building AM is a key part of developing a mathematical model for the research object. During the analysis, all input parameters of the isolated object are sorted out, and using static recognition methods, the model is adjusted in accordance with the purpose. As a result, factors that are not informative for the MM object are excluded. Such theoretical foundations of MM, for example, the theory of elasticity, the theory of plastic deformation and the theory of filtration, were developed in the Central Asian region under the leadership of academicians V.K. Kabulov, F.B. Abutaliev, T. Buriev, D. Fayzullayev and their students. A mathematical model is a representation of the object (process or phenomenon) under study by the student, which is created by the student using certain formal mathematical expressions (systems). It is created for the purpose of research, forecasting, analysis, functionalization and study of the characteristics of complex processes occurring in technical systems. For the development of oil or gas fields, a fixed number of formation parameters is used. The most important of them are: 1) formation pressure and well pressure changes over time; 2) well flows over time; 3) the number of required wells and their changes over time.

These indicators can be determined by secondizing and second-order integration of differential equations for the variable equations of oil or gas filtration in porous media. Due to the complexity and nonlinearity of the two-dimensional oil and gas filtration equations, it is currently impossible to obtain the necessary analytical solutions. Therefore, various numerical methods, computational algorithms and their software have been proposed to calculate the development indicators of oil and gas fields, which are used to conduct computational experiments on computers. In this case, the use of computers and the corresponding effective numerical methods allows obtaining reliable numerical results.

Problem statement

The non-stationary oil filtration process, considered as a one-dimensional problem, is analyzed in porous media. The perfect shape of oil fields, the change in complex parameters of the formation by non-uniform values, the uneven location of wells and different debits (a measure of resource efficiency in oil and gas) cannot be a limitation for the use of numerical modeling. At the same time, taking into account these factors creates difficulties for electronic computers (ECMs) in calculating oil and gas fields with complex configurations and variable hydrodynamic parameters. Many scientists have worked on this issue. In particular, the work of the Uzbek scientist, Candidate of Technical Sciences, Prof. Nazirova. E. Sh. As an example, the study of the filtration processes of oil in a heavily contaminated form showed that when the structure of the wells is intensively operated, fine dispersed particles in the wellbore zones clog the porous media. As a result, the oil yield of the formations decreases. This phenomenon plays an important role in the filtration process of oil in porous media. As a result of studying this problem, a mathematical model was developed that takes into account the following factors: the sedimentation rate of fine dispersed particles, changes in porosity and filtration coefficients over time. The model is used in the effective operation of oil fields and in predicting filtration processes in difficult conditions.

Taking into account the above factors, the non-stationary oil filtration process in an infinitely porous medium is described by the following nonlinear differential equation:

$$\begin{cases} \beta h(x)\dfrac{\partial P}{\partial t}=\dfrac{\partial}{\partial x}\left(\dfrac{k(x)h(x)}{\mu}\dfrac{\partial P}{\partial x}\right)-Q, \\ \dfrac{d\eta}{dt}=\lambda(\theta-\gamma\eta), \end{cases} \quad \text{here } x\in G \quad (1.1)$$

$$m=m_1+\eta(m_0-m_1), k=k_0(1-\sqrt{\eta})^3, \beta=m\beta_H+\beta_c m_0 \qquad (1.2)$$

The differential equation (1.1) is processed under the following conditions.

$$P(x,t)=P_H(x), \ \text{t=0 it is} \qquad (1.3)$$

$$\eta(t)=\eta_0 \ \ \text{t=0 it is} \qquad (1.4)$$

$$\frac{k(x)h(x)}{\mu}\frac{\partial P}{\partial n}=\alpha(P_A-P), \ \text{here } x\in\Gamma \qquad (1.5)$$

$$\oint_{s_{i_q}}\frac{k(x)h(x)}{\mu}\frac{\partial P}{\partial n}ds=-q_{i_q}(t) \ \text{here } (x,y)\in s_{i_q}, i_q=\overline{1,N_q} \qquad (1.6)$$

$$Q=\sum_{i,j=1}^{N_q}\delta_{i,j}q_{i,j}$$

The following variables are included here:

$P-$plate pressure; P_H- initial plate pressure; P_A-boundary pressure; $\mu-$dynamic viscosity of oil; $k-$permeability coefficient of the formation; $h-$layer thickness; $\beta-$elastic capacity coefficient of the formation;

β_H-oil compressibility coefficient; β_c-compressibility coefficient of the medium; $n-$debit in this layer; i_q-contour in this layer; Γ internal normal to this boundary; N_q-number of wells; $m-$ porosity coefficient of the formation;

$m_0 -$ initial porosity; $m_1 -$ porosity of the sedimented part of the mass; $k_0 -$ initial permeability coefficient of the formation; $\eta -$ concentration of finely dispersed particles in a porous medium; $\gamma -$ filtration coefficient; $\lambda -$ kinematic coefficient; $\delta -$ Dirac function,

$$\boldsymbol{\alpha} = \begin{cases} \mathbf{0}, & \boldsymbol{closed\ boundry\ condition}, \\ \mathbf{1}, & \boldsymbol{open\ boundary\ condition} \end{cases}$$

We also take into account the coefficients of permeability porosity (m) and permeability (k) in this equation.

Sedimentation rate of finely dispersed particles $\eta -$ is determined by equation (1.1) of the system of equations.

Solution of the problem

To numerically solve equations (1.1)-(1.6), we pass from the following continuous domain to the discrete domain:

$$P^* = P/P_0; x^* = x/L; k^* = k/k_0; h^* = h/h_0;$$

$$\eta^* = \eta\Big/\eta_0; \tau = \frac{k_0 t}{\beta\mu L^2}; q^* = \frac{q\mu}{\pi k_0 P_0 h_0}; \lambda^* = \frac{\mu L}{k_0 h_0}$$

Here P_0 is the initial pressure; k_0 is the initial permeability of the layer; h_0 is some thickness of the layer, for example, h_0=max(h(x)); L- is the corresponding length.

In the following, for simplicity, we omit in the equations ≪*≫. Thus, we write equations (1.1)-(1.6) in the continuous domain as follows:

$$\begin{cases} h(x)\dfrac{\partial P}{\partial t} = \dfrac{\partial}{\partial x}\left(\dfrac{k(x)h(x)}{\mu}\dfrac{\partial P}{\partial x}\right) - Q, \\ \dfrac{d\eta}{dt} = \lambda(\theta - \gamma\eta), \end{cases} \quad \text{here } x \in G \qquad (1.7)$$

$$\begin{cases} h(x)\dfrac{\partial P}{\partial \tau} = \dfrac{\partial}{\partial x}\left(\dfrac{k(x)h(x)}{\mu}\dfrac{\partial P}{\partial x}\right) - Q, \\ \dfrac{d\eta}{d\tau} = \lambda(\theta - \gamma\eta), \quad here\ x \in G \end{cases} \qquad (1.7)$$

$$m = m_1 + \eta(m_0 - m_1), k = k_0(1 - \sqrt{\eta})^3 \qquad (1.8)$$

$$P(x) = P_H(x) \quad \text{here} \quad t = 0, x \in G \qquad (1.9)$$

$$\eta(t) = \eta_0, t = 0 \qquad (1.10)$$

$$-k(x)h(x)\frac{\partial P}{\partial n} = \alpha(P_A - P) \ \text{ here } x \in \Gamma \qquad (1.11)$$

$$\oint_{s_{i_q}} \frac{k(x)h(x)}{\mu}\frac{\partial P}{\partial n} ds = -q_{i_q}(t) \quad \text{here } x \in s_{i_q}, i_q = \overline{1, N_q}. \qquad (1.12)$$

To solve equations (1.7)-(1.12) numerically, we divide the filtration domain G and its outer domain Γ into squares by one step. $\Delta h = \Delta h_x$:

$$\Omega_{x\tau_k} = \left\{ (x_i = i\Delta h, \tau_k = k\Delta\tau); i = \overline{1, N}, k = \overline{0, N_\tau}, \Delta\tau = \frac{1}{N_\tau} \right\}$$

We assume that each $i_q -$ well corresponds to a grid node with its S_{i_q} own contour.

To obtain the final results, we use the algorithmic idea of each direction (horizontal transverse scheme).

The transition from the t-time layer to the t+1 - layer is carried out in two stages, with $0,5\Delta\tau$ stepwise. As a result, a sequential solution of the system of two final differential equations is obtained. The first final differential equation for the t+0.5 - layer for internal nodes has the following form:

$$\frac{\overline{P}_i^j - P_i^j}{\Delta\tau/2} = \frac{T_{i-0.5}^j P_{i-1}^j - \left(T_{i-0.5}^j + T_{i+0.5}^j\right)\overline{P_i^j} + T_{i+0.5}^j \overline{P_{i+1}^j}}{\Delta h^2} - \delta_i^j q_i^j \qquad (1.13)$$

Here $P_i^j - t -$ time layer pressure value. $\overline{P_i^j} - (t+0.5) -$ pressure value in the time layer.

$$T_i^j = \frac{k_i^j h_i^j}{\mu}; T_{i-0.5}^j = \frac{T_{i-1}^j + T_i^j}{2}; T_{i+0.5}^j = \frac{T_i^j + T_{i+1}^j}{2}; T_{i-0.5}^j = \frac{T_i^{j-1} + T_i^j}{2}; T_i^{j+0.5} = \frac{T_i^j + T_i^{j+1}}{2}$$

To calculate the pressure $\overline{P}_i^j$ value at time t+0.5 with sufficient accuracy, we solve the three-point system of equations for $x = x_i$ the values for each line . Here, k_i^j the value is left unchanged.

$$\begin{cases} (3\gamma - 2\Delta hL\alpha)\overline{P}_0^j - 4\gamma\overline{P_1^j} + \gamma\overline{P_2^j} = -2\Delta hL\alpha P_A, \\ a_i\overline{P_{i-1}^j} - b_i\overline{P_i^j} + c_i\overline{P_{i+1}^j} = -d_i, \\ (3\gamma - 2\Delta hL\alpha)\overline{P_{N_j}^j} - 4\gamma\overline{P_{N_j-1}^j} + \gamma\overline{P_{N_j-2}^j} = 2\Delta hL\alpha P_A \end{cases} \quad \text{here } i = 1, 2, ..., N_j - 1, \; (2.14)$$

Here is the following value:

$$\gamma = \frac{kh}{\mu}, a_i = T_{i-0.5}^j, b_i = T_{i-0.5}^j + T_{i+0.5}^j + \frac{\Delta h^2}{\Delta\tau/2}, c_i = T_{i+0.5}^j,$$

$$d_i = T_i^{j+0.5} P_i^{j-1} - \left(T_i^{j-0.5} + T_i^{j+0.5}\right) P_i^j + T_i^{j+0.5} P_i^{j+1} + \frac{\Delta h^2}{\Delta\tau/2} P_i^j - \delta_i^j q_i^j$$

The equation (1.14) of the system of equations is obtained from the finite difference equations (1.13), and the rest are found from the boundary conditions (1.11), obtained by approximating the boundary of the filtration domain to the second-order accuracy level.

The system (1.14) is solved in steps, here $2 \le j \le M_i - 1$.

Now we write a similar finite difference equation for the time layer t+1:

$$\frac{P_i^j - \overline{P}_i^j}{\Delta\tau/2} = \frac{T_{i-0.5}^j \overline{P}_{i-1}^j - \left(T_{i-0.5}^j + T_{i+0.5}^j\right)\overline{P_i^j} + T_{i+0.5}^j \overline{P_{i+1}^j}}{\Delta h^2} \qquad (1.15)$$

Here, $\overline{P}_i^j -$ the value is the value of the system of equations at time t+0.5, and $P_i^j -$ the value of the system of equations at time t+1.

Based on the finite difference equation (1.15) and the approximation of the boundary conditions (1.11) with respect to h to the second order accuracy, we obtain the following three-point system of equations:

$$\begin{cases} (3\gamma - 2\Delta hL\alpha)P_i^0 - 4\gamma P_i^1 + \gamma P_i^2 = -2\Delta hL\alpha P_A, \\ a_j P_i^{j-1} - b_j P_i^{j+1} + c_i P_i^{j+1} = -d_j, \\ (3\gamma - 2\Delta hL\alpha)P_i^{M_i} - 4\gamma P_i^{M_i-1} + \gamma P_i^{M_i-1} = 2\Delta hL\alpha P_A \end{cases} \quad \text{bu yerda } j = 1, 2, \ldots, M_i - 1, \ (1.16)$$

$$a_j = T_i^{j-0.5}, b_j = T_i^{j-0.5} + T_i^{j+0.5} + \frac{\Delta h^2}{\Delta\tau/2}, c_j = T_i^{j+0.5},$$

$$d_j = T_{i-0.5}^j \overline{P}_i^{j-1} - \left(T_{i-0.5}^j + T_{i+0.5}^j\right)\overline{P}_i^j + T_{i+0.5}^j \overline{P}_{i+1}^j + \frac{\Delta h^2}{\Delta\tau/2}\overline{P}_i^j - \delta_i^j q_i^j$$

(1.16) system of equations $2 \le i \le N_i - 1$ is solved by interval.

Now, the equation of the system (1.1) τ We approximate by:

$$\frac{\eta_i^j - \eta_i^j}{\Delta\tau} = \lambda\left(\theta_0 - \gamma\eta_i^j\right)$$

From this, the rate of sedimentation of finely dispersed particles is η we get it:

$$\eta_i^j = \frac{\lambda\Delta\tau\theta_0 + \eta_i^j}{1 + \lambda\gamma\Delta\tau}$$

Here is η_i^j -the sedimentation rate of particles, η_i^j -which is the sedimentation rate in the current time layer at time t=0, obtained from the initial value (2.4). After determining η_i^j the value, the new porosity (m) and permeability (k) values are calculated in each time layer according to (1.8).

A characteristic feature of systems (1.14) and (1.16) is that they have a three-diagonal matrix for each layer or column. This feature allows us to use the run-in method to find a solution. For this, a system of equations (1.14) and (1.16) is constructed in the directions of conductivity and using the run-in method, and the run-in unknowns are found from the resulting system. First, the run-in unknowns are found along each layer in the Ox direction and an average solution to the problem is obtained for the time layer t+0.5. Then, using the same method, the system of equations (1.16) is constructed and solved. After that, the result is required and the solutions corresponding to the time point t+1 are calculated. The accuracy of these equations is shown.

Conclusion

Considering the oil filtration process, a mathematical model based on several filtration processes was developed, taking into account the sedimentation of fine particles in an infinite environment. Based on the mathematical model of oil filtration in a closed environment, a

IBAST | Volume 4, Issue 12, December

computational algorithm was developed to solve boundary value problems taking into account the sedimentation of fine particles using the longitudinal and transverse schemes and the final finite difference scheme. In this problem, the equation was brought to the three-diagonal run method using the horizontal finite difference method, and the equation was solved using this method and the results were obtained.

The lits of used literatures:

1.Abasov M.T., Jalilov K.N. Voprosy podzemnoy gidrodinamiki i razrabotki neftyanikh i gazovikh mestorozhdeniy. – Baku: Azerneftneshr, 1960. – 255

2.Abutaliev F.B., Nazirova E.Sh. Chislennoye modelirovaniye zadachi filtratsii mnogofaznykh zhidkostey v poristoy srede // Sovremennaya tekhnika i tekhnologii gorno-metallurgicheskoy otrasli i puti ikh razvitiya: Materialy Mezhdunarodnoy nauchno-tekhnicheskoy konferentsii. 12-14 maya 2010. – Navoi, 2010. – S. 456-457.

3.Abutaliev F.B., Khadzhibaev N.N., Izmaylov I.I., Umarov U. Primenenie chislennykh metodov i EVM v gidrogeologii. – Tashkent: Fan, 1976. – 160 s.

4.Abutaliev F.B., Khadzhibaev N.N., Izmaylov I.I., Umarov U. Primenenie chislennykh metodov i EVM v gidrogeologii. – Tashkent: Fan, 1976. – 160 s.

5.Абуталиев Ф.Б., Хаджибаев Н.Н., Измайлов И.И., Умаров У. Применение численных методов и ЭВМ в гидрогеологии. – Ташкент: Фан, 1976. – 160 с.

6.Абуталиев Ф.Б., Кутлумурадов Д., Садуллаев Р. Численно аналитические методы расчета нестационарной фильтрации в системах пластов жидкости и газа. – Нукус: Каракалпакстан, 1977. – 232 с.

7.“МАТЕМАТИЧЕСКИЕ МОДЕЛИ, ЧИСЛЕННЫЕ МЕТОДЫ И КОМПЛЕКСЫ ПРОГРАММ ДЛЯ ИССЛЕДОВАНИЯ ПРОЦЕССОВ ФИЛЬТРАЦИИ ЖИДКОСТЕЙ И ГАЗОВ” dessertatsiya. Texnika fanlari nomzodi prof. Nazirova.E.Sh Toshkent 2019. 33-63-s