APPLICATIONS OF INTEGRATION TO BUSINESS AND ECONOMICS

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ABSTRACT. Just like the process of differentiation is a useful tool in many business and economics applications such as problems related to elasticity of demand and optimization, so is the process of *anti*differentiation or integration. In this section, we revisit two important business and economic models, namely concerning law of supply and demand in a free-market environment and continuous money flow, and introduce integration as a method for solving problems of this nature.

KEYWORDS: *Integration*, *Interactive Demonstration***,** Surplus**,** *equilibrium, Cobb-Duglas theorem,*

INTRODUCTION

Recall that in a free market, the consumer demand for a particular commodity is dependent on the commodity's unit price, which is captured graphically by the demand curve. As expected, the quantity demanded of a commodity increases as the commodity's unit price decreases, and vice versa. Similarly, the unit price of a commodity is dependent on the commodity's availability in the market, which is articulated graphically by the supply curve. Typically, an increase or decrease in the commodity's unit price induces the producer to respectively increase or decrease the supply of the commodity. Below we see the supply and demand curves of a certain item produced and sold:

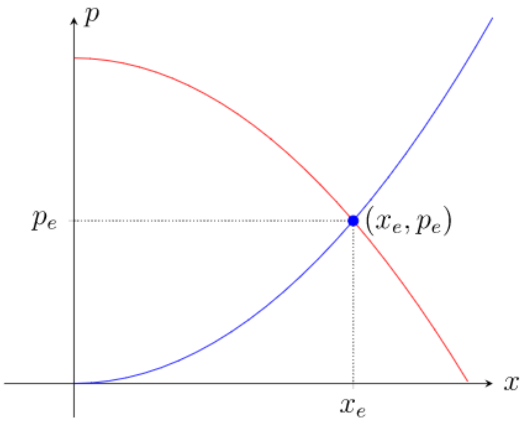
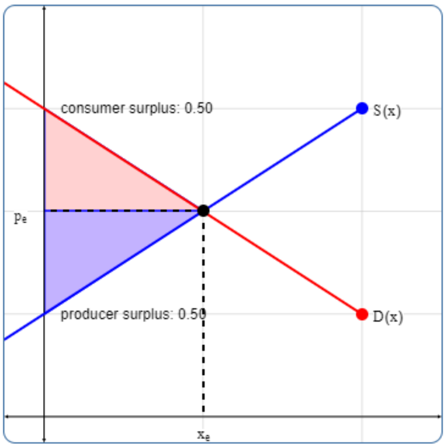


Figure 3.1. Example of a supply curve (in blue) and a demand curve (in red). The point of intersection $(x_e\ p_e)$ corresponds to market equilibrium.

METHODS . ***Interactive Demonstration.*** Use the control points below to change the producer and supplier surpluses (the equilibrium point is fixed).



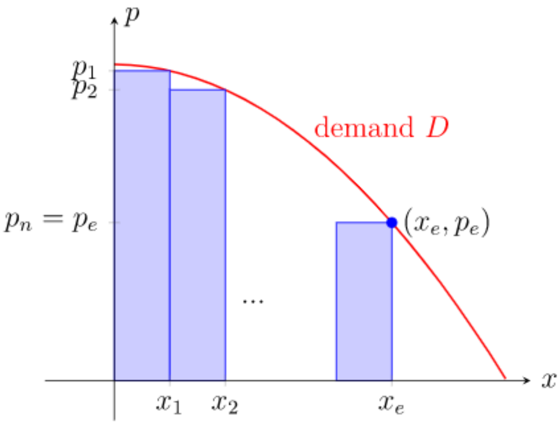
In a competitive market, the price of a commodity will eventually settle at the market equilibrium, which occurs when the supply of the commodity will be equal to its demand as indicated with the point (x_e p_e) in figure 3.6. Let us look at a certain commodity, say, dairy to discuss the economic significance of the market equilibrium. As long as $p < p_e$, then the demand for dairy exceeds its supply (see figure 3.6), which pushes up the price until it reaches the equilibrium price p_e, which in turn signifies that the quantity supplied is equal to the quantity demanded, namely x_e. On the other hand, if $p > p_e$, then the supply of dairy exceeds demand (see figure 3.6), which brings the price down. In an ideal free market, buying and selling at the equilibrium price should benefit both consumers and producers. In this section, we will compute the ***surplus***, which tells us exactly how much the consumers save and the producers gain by buying and selling respectively at the equilibrium price rather than at a higher price.

We begin by computing exactly how much consumers spent when they buy at the equilibrium price p_e :

total amount spent at equilibrium price =(number of units bought at equilibrium price)· (unit price)= $x_e \cdot p_e$***total amount spent at equilibrium price =(number of units bought at equilibrium price)· (unit price)=*** $x_e \cdot p_e$

The quantity $x_e \cdot p_e$is the rectangular area shown in figure 3(a).

Now we compute the total amount that would be spent if every consumer paid the maximum price that each is willing to pay. Given a demand function D, partition the interval $[0, x_e]$ into n subintervals of equal width $\Delta x = \frac{x_e}{n}$ with endpoints $x_i = \frac{ix_e}{n}, i = 1,2,3, \dots n$ as shown below:



Now, let us analyze this partition subinterval by subinterval. Suppose that only x_1 units had been available, then the maximum unit price could have been set at $D(x_1)$ dollars and a total of x_1 units would be sold, but at this price no further units would have been sold. Then the total expenditure in dollars is given by

(price per unit)· (number of units)=$D(x_1)$· Δx.

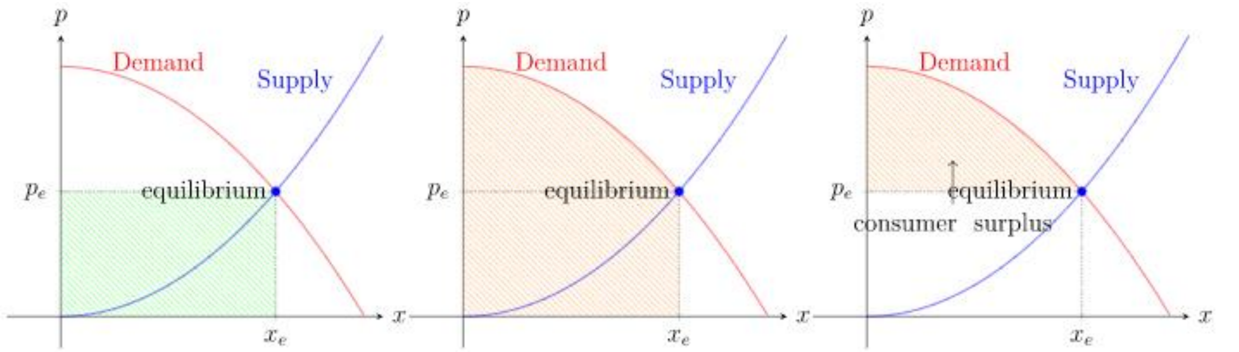
Now suppose that more units become available by producing x_2 units of our commodity. If the maximum price is set at $D(x_2)$ dollars, then the remaining $x_2 - x_1 = \Delta x$ units can be sold at a cost of $D(x_2)\Delta x$ dollars. If we continue with this process of price discrimination, then the total amount spent at maximum price is approximately equal to

$$D(x_1)\cdot\Delta x + D(x_2)\cdot\Delta x + \cdots + D(x_n)\cdot\Delta x.$$

Note:

1. On each of the subintervals $[0, x_1], [x_1, x_2], \dots, [x_{n-1}, x_e]$, the buyers paid as much for each unit as it was worth to them.
2. We recognize that the last sum is a Riemann sum, which yields $\int_0^{x_e} D(x)dx$ as $n \to \infty$ or alternatively as $\Delta x \to 0$.

Of course, we simply have calculated the area under the demand curve on the interval $[0, x_e]$, which is shown in *figure 3 (b).*



(a) (b) (c)

Figure 3.2.

The difference between the total amount spent at the maximum price (figure 3.(b)) and the consumer expenditure at the equilibrium price (figure 3. (a)) is the total amount that consumers save by buying at the equilibrium price (figure 3.(c)). This saving is called the ***consumer surplus*** for this product. We summarize our result.

RESULTS AND DISCUSSION

Theorem 1. Consumer Surplus.

Let D be the demand function, p_e be the equilibrium price, and x_e be the equilibrium quantity sold. Then the consumer surplus is

$$\int_0^{x_e} D(x)dx - x_e\cdot p_e = \int_0^{x_e} [D(x) - p_e]dx.$$

In a similar manner, we can determine how much producers gain when they sell at the equilibrium price p_e:

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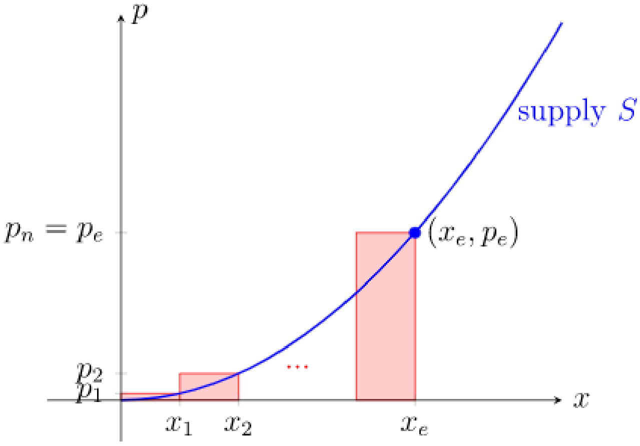
total amount gained at equilibrium price =(number of units sold at equilibrium price)· (unit price)= $x_e \cdot p_e$.

Now we compute the total amount that would be gained if every producer sold at the minimum amount they are willing to accept for the product. Given a supply function *S*, partition the interval $[0, x_e]$ into nn subintervals of equal width $\Delta x = \frac{x_e}{n}$ with endpoints $x_i = \frac{ix_e}{n}, i = 1,2,3, \dots n$ as shown below:



Similarly to the subinterval by subinterval analysis for the demand curve, an analysis of the partition of the supply curve shows that the total revenue on every subinterval in dollars by selling at the minimum price is given by
(price per unit) · (number of units)= $S(x_i)\Delta x$

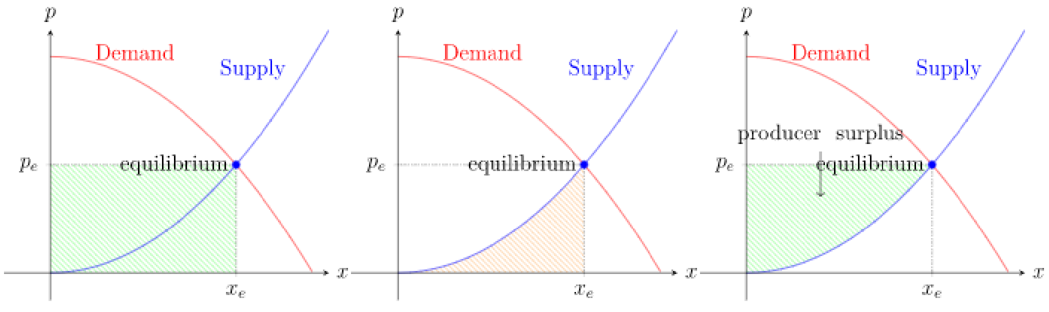
for *i=1,2,…,n*. Continuing with the process of price discrimination, the total amount gained at minimum price is approximately equal to

$$S(x_1) \cdot \Delta x + S(x_2) \cdot \Delta x + \cdots + S(x_n) \cdot \Delta x$$

Note:

1. On each of the subintervals $[0, x_1], [x_1, x_2], \dots, [x_{n-1}, x_e]$, the producers sold at the lowest price that they are willing to set.
2. This sum is again a Riemann sum, which yields $\int_0^{x_e} D(x)dx$ as $n \to \infty$ or alternatively as $\Delta x \to 0$.

Figure 3.(b) shows the area under the supply curve on the interval $[0, x_e]$ that our computation has yielded.



(a) (b) (c)

Figure3.3. Producer revenue on $[0, x_e]$ at equilibrium price, in total, and for surplus.

The difference between the producer revenue at the equilibrium price (figure 3.(a)) and the total amount achieved at the minimum price (figure 3.(b)) is the total amount that producers gain by selling at the equilibrium price (figure 3.(a)). This income is called the ***producer surplus*** for this product. We summarize our result.

Theorem 2. Producer Surplus.

Let S be the supply function, p_e be the equilibrium price, and x_e be the equilibrium quantity sold. Then the producer surplus is

$$x_e \cdot p_e - \int_0^{x_e} S(x)dx = \int_0^{x_e} [p_e - S(x)]dx.$$

Note: In general, the consumer surplus and the producer surplus are not equal.

Example

The demand for a product, in dollars, is

$$D(x) = 1000 - 0.5x - 0.0002x^2$$

Find the consumer surplus when the sales level is 200.

Solution

When the number of units sold is $x_e = 200$, the corrisponding prise is

$$p_e = 1000 - 0.5 \cdot 200 - 0.0002 \cdot 200^2$$

Therefore, the consumer surplus is

$$\int_0^{200} [D(x) - p_e]dx = \int_0^{200} (1000 - 0.5x - 0.0002x^2 - 892)dx$$

$$= \int_0^{200} (108 - 0.5x - 0.0002x^2)dx = \left[108x - 0.25x^2 - \frac{0.0002}{3}x^3\right]_0^{200}$$

$$= \$\mathbf{11,066.70}.$$

CONCLUTIONS

We recommend to avoid complexity as much as possible in describing the topic, and to pass on topics suitable for specialization without deepening. It is not recommended to consider various theorems with complex proofs. If each subject is oriented to the profession in accordance with the specialties being taught, we will make a worthy contribution to the

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harmony of science and production. In addition, motivation for the profession is formed in the students. They understand that the fundamental sciences they are studying are the basis of production, techniques and technologies.

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