
# LAVRENTOV'S METHOD FOR KARLEMAN'S FORMULA.

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**Annotatsiya.** Ma'lumki golomorf funksiyalar uchun muhum formulalardan biri Koshining integral formulasidir. Bu formula orqali chegarada berilgan funksiyani soha ichida golomorf tiklash mumkin. Karleman formulalarida esa chegaraning qismida berilgan funksiyani soha ichida golomorf tiklash masalasi qaraladi. Karleman formulasini hisoblashning bir nechta usullari mavjud bo'lib xususan Goluzin-Krilov, Kitminov,Lavrentov usullari va hakazo.

**Annotation**. One of the important formulas for know holomorphic functions is is Couchy"s integral formula. Using this formula, the function given at the boundary can be restored to the holomorphic within the field. Carleman formulas deal with te problem the holomorphic restoration of a given function within a boundary there are several methods of calculatinhg the Carleman formula,in particular Goluzin-Krilov, Kitminov,Lavrentov methods and soon.

It is known that one of the important formulas for holomorphic functions is Cauchy's integral formula. Using this formula, the function given on the boundary can be restored holomorphically within the domain. In Karleman's formulas, the problem of holomorphic restoration of the given function in the domain is considered. There are several methods of calculating Karleman's formula, namely Goluzin-Krylov, Kitminov, Lavrentov methods and Hakazo. But the Lavrentov method is more perfect than the Goluzin-Krylov, Kitminov methods of constructing the Karleman formula.

Let us approximate the Cauchy kernel $g_{z,m}(\xi)$ (holomorphic and bounded in the field $D$) on the set $\frac{1}{2\pi i}\frac{1}{\xi - z}$ (here fixed as $z\epsilon D$). We consider this function $\partial D \backslash M$ to be integrable in $\partial D$ with the orthogonal holomorphic function $f \in H^1(D)$). Besides

$$\lim_{m\to\infty} \int_{\partial D \backslash M} f(\xi)[\frac{1}{2\pi i}\frac{1}{t-z} - g_{z,m}(\xi)]d\xi = 0$$

be, then we have the Carleman formula as follows.

$$f(z) = \lim_{m\to\infty} \int_{M} f(\xi)[\frac{1}{2\pi i}\frac{1}{t-z} - g_{z,m}(\xi)]d\xi$$

The kernel under integral formula 1 is holomorphic with respect to parameter $z$, and the integral formula with holomorphic kernel is formed in formula 2.

**Theorem.** 1. Let $D$ be a bounded domain whose boundary consists of a smooth closed Jordan curve with a finite number of segments, and let M be an open partial set in ∂D. Then there is a Karleman formula for the function $f \in H(D)$ whose kernel is holomorphic with respect to $z\epsilon D$. This formula is constructed using a chain of integrals and series expansion.

**Proof.** According to Runge's theorem, $\partial D \backslash M$ -compact is convex compact for sufficiently small $\varepsilon > 0$. We consider $A(D_\varepsilon)$ the sequence of $K_m$compacts satisfying the following conditions:

1. $K_m, m = 1,2 \ldots K_{m+1} \supset K_m$,
2. $\cup_m K_m = D$,
3. Every $K_m$lar A(D_e) is $A(D_\varepsilon)$ convex.

A clear function $\frac{1}{2\pi i}\frac{1}{\xi - z}$is holomorphic in $\partial D\backslash M \times K_m$ any function from the class $A(\partial D\backslash M \times K_m)$ is compact in $A(D_\varepsilon * D_\varepsilon)$ $(\partial D\backslash M) \times K_m$smoothly approximates the function belonging to the class. Indeed, the $A(D_\varepsilon)$ -convexity of $(\partial D\backslash M)$and $K_m$ leads to the $A(D_\varepsilon * D_\varepsilon)$-convexity of $(\partial D\backslash M) \times K_m$. Bu tasdiqni Koshining ikki karrali integral formulasidan $(\partial D\backslash M) \times K_m$ sohaning atrofida qo‘llab va integralni integral yig‘indilarning limiti ekanligidan foydalanib osongina isbotlash mumkin. So,

$\lim_{n\to\infty} f_{m,n}(\xi, z) = \frac{1}{2\pi i}\frac{1}{\xi - z}((\partial D\backslash M) \times K_m)$

where is a smooth convergent at

$f_{m,n}(\xi, z) \epsilon A(D_\varepsilon * D_\varepsilon), g_{z,m}(\xi) = f_{m,n(n)}(\xi, z)$

if we assume that $n = n(m)m$ is a fixed number, the following will be appropriate in the considered compact:

$$\left|\frac{1}{2\pi i}\frac{1}{\xi - z} - f_{m,n}(\xi, z)\right| < \frac{1}{m}$$

The function $g_{z,m}(\xi)$is holomorphic in $D_\varepsilon * D_\varepsilon$ and the Karleman formula is valid.

Functions of complex variables $z, \xi$ and $\alpha$ that is, the function denoted as $G(z, t, \alpha)$ is called Karleman function in the set $M$ in the field D, if it fulfills the following conditions:

1. $G(z, \xi, \alpha) = \frac{1}{t - z} + G(\widetilde{z, \xi}, \alpha)$ where $\tilde{G}$ is a holomorphic and bounded function in $D$ space with respect to the function $\xi$,
2. $\frac{1}{2\pi i}\int_{\partial D\backslash M} |G(z, \xi, \alpha)||d\xi| \leq C(z)\alpha$, where $C(z)$ depends on the constant variable $z$.

The kernel in the Karleman formula is an example of a Karleman function.

$$G(z, \xi, \alpha) = \left[\frac{\varphi(\xi)}{\varphi(z)}\right]^{\frac{1}{\alpha}}\frac{1}{\xi - z}$$

In general, if the Karleman function $G$ is given, then the Karleman formula is as follows

$$f(z) = \lim_{\alpha\to 0}\frac{1}{2\pi i}\int_M f(\xi)G(z, \xi, \alpha)$$

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