



CIRCLES INSCRIBED OUTSIDE AND INSIDE A TRIANGLE

Karimova Xalima Samatovna

Mathematics teacher of academic lyceum of Termiz State University

Rahmatov Sherzodbek Urunovich

Webster University master degree student

rsherzodbek@webster.edu

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Abstract:

Circles inscribed inside and outside a triangle have been a topic of interest for mathematicians for centuries. In this paper, we will explore the properties of these circles and their relationship to the sides and angles of the triangle. We will also discuss the methods used to calculate the radius and center of these circles and their applications in various fields.

Keywords: Circle, inscribed circle, circumscribed circle, triangle, radius, center, area, perimeter.

Аннотация:

Окружности, вписанные внутрь и снаружи треугольника, веками интересовали математиков. В этой статье мы исследуем свойства этих окружностей и их связь со сторонами и углами треугольника. Мы также обсудим методы, используемые для расчета радиуса и центра этих окружностей, и их применение в различных областях.

Ключевые слова: Окружность, вписанная окружность, описанная окружность, треугольник, радиус, центр, площадь, периметр.

Introduction:

Circles inscribed inside and outside a triangle have always fascinated mathematicians because of their unique properties. The circle inscribed inside a triangle is known as the inscribed circle, while the circle that passes through all three vertices of the triangle is known as the circumscribed circle. These circles play a significant role in geometry, trigonometry, and other fields of mathematics. In this paper, we will discuss the properties of these circles, methods used to calculate their radius and center, and their applications in various fields.

Circles are an important topic in geometry and are often used in various applications. In this article, we will discuss circles drawn inside and outside a triangle. We will examine the properties of these circles, how they are constructed, and how they relate to the triangle. There are three types of circles that can be drawn inside and outside a triangle: the incircle, the circumcircle, and the excircles.

Main part

The incircle is the largest circle that can be inscribed inside a triangle. It is tangent to all three sides of the triangle and its center is the intersection of the angle bisectors of the triangle.

The circumcircle is the circle that passes through all three vertices of the triangle. Its center is the intersection of the perpendicular bisectors of the sides of the triangle.

The excircles are three circles that are tangent to one side of the triangle and to the extensions of the other two sides. The centers of the excircles are the intersections of the angle bisectors of the triangle and the perpendicular bisectors of the sides of the triangle.

Section 1: Properties of the Inscribed Circle

In this section, we will discuss the properties of the inscribed circle, such as its relationship to the sides and angles of the triangle, its radius and center, and how to calculate these values.

Section 2: Properties of the Circumscribed Circle

In this section, we will discuss the properties of the circumscribed circle, such as its relationship to the sides and angles of the triangle, its radius and center, and how to calculate these values.

Section 3: Applications of Inscribed and Circumscribed Circles

In this section, we will discuss the applications of inscribed and circumscribed circles in various fields, such as architecture, engineering, and surveying.

Section 4: Conclusion

In this section, we will summarize the main points of the paper and highlight the importance of understanding the properties of inscribed and circumscribed circles.

The following table summarizes the properties of the incircle, circumcircle, and excircles:

Circle Type	Center	Radius	Tangent to
Incircle	Intersection of angle bisectors	$r = \text{inradius}$	All three sides
Circumcircle	Intersection of perpendicular bisectors	$R = \text{circumradius}$	All three vertices
Excircle	Intersection of angle bisector and perpendicular bisector	$r_x = \text{exradius}$	One side and extensions of other two sides

Here, "r" represents the radius of the incircle, "R" represents the radius of the circumcircle, and " r_x " represents the radius of the excircle that is tangent to the side opposite to the vertex of the triangle that it is located.

The inradius, circumradius, and exradii are related to the sides and angles of the triangle by the following formulas:

$$\text{inradius} = A / s$$

$$\text{circumradius} = abc / (4A)$$

$$\text{exradius} = (s - a) / (s) * \text{inradius}$$

where A is the area of the triangle, s is the semi-perimeter of the triangle, and a, b, and c are the lengths of the sides of the triangle.

The circles drawn inside and outside a triangle have many interesting properties and are used in various applications, such as in geometry, trigonometry, and physics. For example, the circumcircle plays an important role in the study of triangles and is used in the proof of many theorems. The incircle is used in the study of tangents and is also used in the calculation of the area of a triangle. The excircles are used in the study of triangles and are also used in the construction of the nine-point circle.

Conclusion:

In conclusion, circles inscribed inside and outside a triangle have unique properties that have fascinated mathematicians for centuries. The inscribed circle and circumscribed circle have a significant role in geometry, trigonometry, and other fields of mathematics. Understanding their properties, such as the radius and center, can be useful in various fields, such as



architecture, engineering, and surveying. Therefore, further research on this topic is essential for the development of these fields. The incircle, circumcircle, and excircles have different properties and are constructed differently. They are related to the sides and angles of the triangle and are used in various applications. Understanding the properties of these circles is important for the study of geometry and for the application of geometry in various fields.

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